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UNIVERSITY OF NORTH BENGAL

MASTER OF ARTS- PHILOSOPHY

SEMESTER –II

WESTERN LOGIC

CORE-201

BLOCK-2

UNIVERSITY OF NORTH BENGAL

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ENLIGHTENMENT TO PERFECTION

FOREWORD

The Self Learning Material (SLM) is written with the aim of providing simple and organized study content to all the learners. The SLMs are prepared on the framework of being mutually cohesive, internally consistent and structured as per the university's syllabi. It is a humble attempt to give glimpses of the various approaches and dimensions to the topic of study and to kindle the learner's interest to the subject

We have tried to put together information from various sources into this book that has been written in an engaging style with interesting and relevant examples. It introduces you to the insights of subject concepts and theories and presents them in a way that is easy to understand and comprehend.

We always believe in continuous improvement and would periodically update the content in the very interest of the learners. It may be added that despite enormous efforts and coordination, there is every possibility for some omission or inadequacy in few areas or topics, which would definitely be rectified in future.

We hope you enjoy learning from this book and the experience truly enrich your learning and help you to advance in your career and future endeavours.

WESTERN LOGIC

BLOCK 1

BLOCK 2

BLOCK 2 : WESTERN LOGIC

Introduction to the Block

Unit 8 deals salient features of syllogism, which forms an important part of classical or Aristotelian Syllogism

Unit 9 deals with inference is the main subject matter of logic

Unit 10 deals with new set of rules to test the validity of arguments, which consist of general and singular propositions

Unit 11 deals with expose the inadequacy of the rules of inference. While this is the primary objective, which is intended to be achieved, there is another objective.

Unit 12 deals with propose to introduce a new list of techniques of testing the validity of arguments

Unit 13 deals with Attributes of relations

Unit 14 deals with primary aim of this section is to assemble a few facts that we will need in the rest of the course.

UNIT 8: SYLLOGISM

STRUCTURE

8.0 OBJECTIVES

In this unit an attempt is made:

- To introduce to you salient features of syllogism, which forms an important part of classical or Aristotelian Syllogism.

- To integrate traditional analysis with modern analysis. In doing so, some vital differences between these analyses are brought to the fore.

8.1 INTRODUCTION

Syllogism is the most important part of Aristotle's logic. It is a kind of mediate inference in which conclusion follows from two premises. We consider two kinds of syllogism, viz., conditional and unconditional. Further, under conditional, there are two divisions: mixed and pure. We can consider conditional syllogism at a later stage. In this unit, we shall confine ourselves to unconditional syllogism or categorical syllogism.

In antiquity, two rival theories of the syllogism existed: Aristotelian syllogistic and Stoic syllogistic. Aristotle defines the syllogism as "a discourse in which certain (specific) things having been supposed, something different from the things supposed results of necessity because these things are so". Despite this very general definition, in Prior Analytics Aristotle limits himself to categorical syllogisms that consist of three categorical propositions. These include categorical modal syllogisms.

From the Middle Ages onwards, categorical syllogism and syllogism were usually used interchangeably. This article is concerned only with this traditional use. The syllogism was at the core of traditional deductive reasoning, where facts are determined by combining existing statements, in contrast to inductive reasoning where facts are determined by repeated observations.

Within academic contexts, the syllogism was superseded by first-order predicate logic following the work of Gottlob Frege, in particular his Begriffsschrift (Concept Script) (1879), but syllogisms remain useful in some circumstances, and for general-audience introductions to logic.

A categorical syllogism consists of three parts:

- Major premise
- Minor premise
- Conclusion

Each part is a categorical proposition, and each categorical proposition contains two categorical terms. In Aristotle, each of the premises is in the form "All A are B," "Some A are B", "No A are B" or "Some A are not B", where "A" is one term and "B" is another. "All A are B," and "No A are B" are termed *universal* propositions; "Some A are B" and "Some A are not B" are termed *particular* propositions. More modern logicians allow some variation. Each of the premises has one term in common with the conclusion: in a major premise, this is the *major term* (*i.e.*, the predicate of the conclusion); in a minor premise, this is the *minor term* (*i.e.*, the subject of the conclusion). For example:

Major premise: All humans are mortal.

Minor premise: All Greeks are humans.

Conclusion: All Greeks are mortal.

Each of the three distinct terms represents a category. In the above example, *humans*, *mortal*, and *Greeks*. *Mortal* is the major term, *Greeks* the minor term. The premises also have one term in common with each other, which is known as the *middle term*; in this example, *humans*. Both of the premises are universal, as is the conclusion.

Major premise: All mortals die.

Minor premise: All men are mortals.

Conclusion: All men die.

Here, the major term is *die*, the minor term is *men*, and the middle term is *mortals*. Again, both premises are universal, hence so is the conclusion.

A sorites is a form of argument in which a series of incomplete syllogisms is so arranged that the predicate of each premise forms the subject of the next until the subject of the first is joined with the predicate of the last in the conclusion. For example, one might argue that all lions are big cats, all big cats are predators, and all predators are carnivores. To conclude that therefore all lions are carnivores is to construct a sorites argument.

There are infinitely many possible syllogisms, but only 256 logically distinct types and only 24 valid types (enumerated below). A syllogism takes the form:

Major premise: All M are P.

Minor premise: All S are M.

Conclusion: All S are P.

(Note: M – Middle, S – subject, P – predicate. See below for more detailed explanation.)

The premises and conclusion of a syllogism can be any of four types, which are labeled by letters as follows. The meaning of the letters is given by the table:

code	***quantifier***	***subject***	***copula***	***predicate***	***type***	***Example***
A	All	S	are	P	universal affirmative	All humans are mortal.
E	No	S	are	P	universal negative	No humans are perfect.
I	Some	S	are	P	particular affirmative	Some humans are healthy.
O	Some	S	are not	P	particular negative	Some humans are not clever.

In *Analytics*, Aristotle mostly uses the letters A, B, and C (the Greek letters alpha, beta, and gamma in the original) as term place holders, rather than giving concrete examples. It is traditional to use *is* rather than *are* as the copula, hence *All A is B* rather than *All As are Bs*. It is traditional and convenient practice to use a, e, i, o as infix operators so the categorical statements can be written succinctly. The following table shows the longer form, the succinct shorthand, and equivalent expressions in predicate logic:

Form	**Shorthand**	**Predicate logic**
All A is B	AaB	*or*
No A is B	AeB	*or*
Some A is B	AiB	
Some A is not B	AoB	

The convention here is that the letter S is the subject of the conclusion, P is the predicate of the conclusion, and M is the middle term. The major premise links M with P and the minor premise links M with S. However, the middle term can be either the subject or the predicate of each premise where it appears. The differing positions of the major, minor, and middle terms gives rise to another classification of syllogisms known as the *figure*. Given that in each case the conclusion is S-P, the four figures are:

	Figure 1	***Figure 2***	***Figure 3***	***Figure 4***
Major premise	M–P	P–M	M–P	P–M
Minor premise	S–M	S–M	M–S	M–S

(Note, however, that, following Aristotle's treatment of the figures, some logicians—e.g., Peter Abelard and John Buridan—reject the fourth figure as a figure distinct from the first. See entry on the Prior Analytics.)

Putting it all together, there are 256 possible types of syllogisms (or 512 if the order of the major and minor premises is changed, though this makes no difference logically). Each premise and the conclusion can be of type A, E, I or O, and the syllogism can be any of the four figures. A syllogism can be described briefly by giving the letters for the premises and conclusion followed by the number for the figure. For example, the syllogism BARBARA below is AAA-1, or "A-A-A in the first figure".

The vast majority of the 256 possible forms of syllogism are invalid (the conclusion does not follow logically from the premises). The table below shows the valid forms. Even some of these are sometimes considered to commit the existential fallacy, meaning they are invalid if they mention an empty category. These controversial patterns are marked in *italics*. All but four of the patterns in italics (felapton, darapti, fesapo and bamalip) are weakened moods, i.e. it is possible to draw a stronger conclusion from the premises.

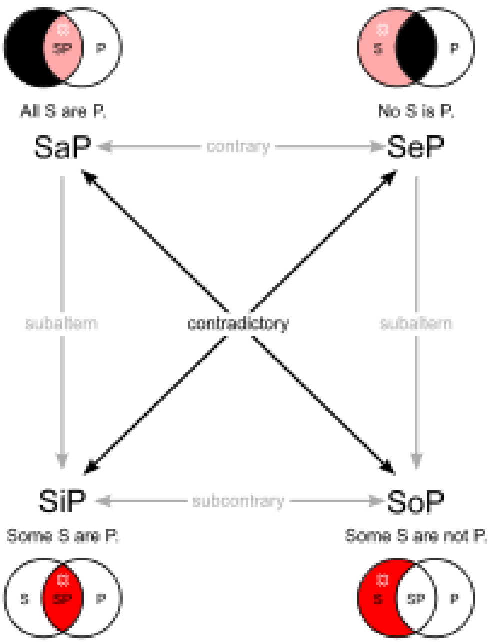
Figure 1	***Figure 2***	***Figure 3***	***Figure 4***
Barbara	Cesare	Datisi	Calemes
Celarent	Camestres	Disamis	Dimatis
Darii	Festino	Ferison	Fresison
Ferio	Baroco	Bocardo	*Calemos*
Barbari	*Cesaro*	*Felapton*	*Fesapo*
Celaront	*Camestros*	*Darapti*	*Bamalip*

The letters A, E, I, and O have been used since the medieval Schools to form mnemonic names for the forms as follows: 'Barbara' stands for AAA, 'Celarent' for EAE, etc.

Next to each premise and conclusion is a shorthand description of the sentence. So in AAI-3, the premise "All squares are rectangles" becomes "MaP"; the symbols mean that the first term ("square") is the middle term, the second term ("rectangle") is the predicate of the conclusion, and the relationship between the two terms is labeled "a" (All M are P).

The following table shows all syllogisms that are essentially different. The similar syllogisms share the same premises, just written in a different way. For example "Some pets are kittens" (SiM in Darii) could also be written as "Some kittens are pets" (MiS in Datisi).

In the Venn diagrams, the black areas indicate no elements, and the red areas indicate at least one element. In the predicate logic expressions, a horizontal bar over an expression means to negate ("logical not") the result of that expression.



We may, with Aristotle, distinguish singular terms, such as Socrates, and general terms, such as Greeks. Aristotle further distinguished (a) terms that could be the subject of predication, and (b) terms that could be predicated of others by the use of the copula ("is a"). (Such a predication is known as a distributive as opposed to non-distributive as in Greeks are numerous. It is clear that Aristotle's syllogism works only for distributive predication for we cannot reason All Greeks are animals, animals are numerous, therefore All Greeks are numerous.) In Aristotle's view singular terms were of type (a) and general terms of type (b). Thus Men can be predicated of Socrates but Socrates cannot be predicated of anything. Therefore, for a term to be interchangeable—to be either in the subject or predicate position of a proposition in a syllogism—the terms must be general terms, or categorical terms as they came to be called. Consequently, the propositions of a syllogism should be categorical propositions (both terms general) and syllogisms that employ only categorical terms came to be called categorical syllogisms.

It is clear that nothing would prevent a singular term occurring in a syllogism—so long as it was always in the subject position—however, such a syllogism, even if valid, is not a categorical syllogism. An example is Socrates is a man, all men are mortal, and therefore Socrates is mortal. Intuitively this is as valid as All Greeks are men, all men are mortal therefore all Greeks are mortals. To argue that its validity can be explained by the theory of syllogism would require that we show that Socrates is a man is the equivalent of a categorical proposition. It can be argued Socrates is a man is equivalent to All that are identical to Socrates are men, so our non-categorical syllogism can be justified by use of the equivalence above and then citing BARBARA

8.2 THE STRUCTURE OF CATEGORICAL SYLLOGISM

For the time being, let us assume that syllogism means valid categorical syllogism unless otherwise qualified. Syllogism consists of two premises and a conclusion. Thus, we have three prepositions and only three terms. An argument is not syllogistic at all unless it conforms to this structure. Since the number of propositions and terms is three, it is quite obvious that every term occurs twice. Consider an example for a syllogistic argument. 1st premise: All humans are stupid. 2nd premise: All sages are human. Conclusion: Therefore all sages are stupid. A term, which is common to the premises (human), is called middle (M). Predicate of the conclusion (stupid) is called major (P) and subject of the conclusion (sages) is called minor (S). While major has maximum extension, minor has minimum extension. The middle term is so called because its extension varies between the limits set by minor and major. The premise in which major occurs is called major premise and the premise in which minor occurs is called minor premise. Though in this argument the first premise is major and the second is minor there is no rule which stipulates that this must be the order. Not only can minor premise be written first, but also the conclusion can as well be the first statement. The only restriction is that if an argument starts with premises, always ‘therefore’ or its synonym must precede the conclusion and if the conclusion is the starting point, then ‘because’ or its synonym must be immediately follow the conclusion. Aristotle argued that our inference proceeds from minor term to major term through middle term. Therefore in the absence of middle term, it is impossible to proceed from minor to major. Aristotle is also a pioneer who discovered predicate logic. He restricted syllogism to subject-predicate logic and, naturally he did not give credence to other forms of proposition like relational prepositions. Most of what Aristotle said on syllogism holds good only when we consider predicate logic.

8.3 AXIOMS OF SYLLOGISM

There are two types of axioms: axioms of quantity and axioms of quality. Rules under these axioms are merely stated because there is no proof to these rules.

A. Axioms of Quantity:

A1: The middle must be distributed at least once in the premise.

A2: A term, which is undistributed in the premise, must remain undistributed in the
conclusion. A term, which is distributed in the conclusion, should compulsorily be
distributed in the premise.

B. Axioms of quality:

B1: Two negative premises do not yield any conclusion.

B2: Affirmative premises yield only affirmative conclusion.

B3: Negative premise (there can be only one negative premise) yields only negative conclusion.

Three corollaries follow from these rules. They are as follows: -

1. The number of terms distributed in the conclusion must be one less than the number of terms distributed in the premises. It is very easy to explain this corollary. The number of terms in the conclusion itself is one less than the number of terms in the premises and M which is compulsorily distributed in the premises is not a part of the conclusion. 2. Two particular premises do not yield any conclusion. Only one particular premise is permissible. 3. Particular premise yield only particular conclusion. [The reader is advised to prove these corollaries with the help of Axioms of quality and quantity.]

If a statement includes a term such that the statement is false if the term has no instances, then the statement is said to have existential import with respect to that term. It is ambiguous whether or not a universal statement of the form *All A is B* is to be considered as true, false, or even meaningless if there are no As. If it is considered as false in such cases, then the statement *All A is B* has existential import with respect to A.

It is claimed Aristotle's logic system does not cover cases where there are no instances. Aristotle's goal was to develop "a companion-logic for science. He relegates fictions, such as mermaids and unicorns, to the realms of poetry and literature. In his mind, they exist outside the ambit of science. This is why he leaves no room for such non-existent entities in his logic. This is a thoughtful choice, not an inadvertent omission. Technically, Aristotelian science is a search for definitions, where a definition is 'a phrase signifying a thing's essence.'... Because non-existent entities cannot be anything, they do not, in Aristotle's mind, possess an essence... This is why he leaves no place for fictional entities like goat-stags (or unicorns)." [13] However, many logic systems developed since *do* consider the case where there may be no instances.

However, medieval logicians were aware of the problem of existential import and maintained that negative propositions do not carry existential import, and that positive propositions with subjects that do not supposit are false.

The following problems arise:

(a) In natural language and normal use, which statements of the forms All A is B, No A is B, Some A is B and Some A is not B have existential import and with respect to which terms?
(b) In the four forms of categorical statements used in syllogism, which statements of the form AaB, AeB, AiB and AoB have existential import and with respect to which terms?
(c) What existential imports must the forms AaB, AeB, AiB and AoB have for the square of opposition to be valid?
(d) What existential imports must the forms AaB, AeB, AiB and AoB have to preserve the validity of the traditionally valid forms of syllogisms?

(e) Are the existential imports required to satisfy (d) above such that the normal uses in natural languages of the forms All A is B, No A is B, Some A is B and Some A is not B are intuitively and fairly reflected by the categorical statements of forms AaB, AeB, AiB and AoB?

For example, if it is accepted that AiB is false if there are no As and AaB entails AiB, then AiB has existential import with respect to A, and so does AaB. Further, if it is accepted that AiB entails BiA, then AiB and AaB have existential import with respect to B as well. Similarly, if AoB is false if there are no As, and AeB entails AoB, and AeB entails BeA (which in turn entails BoA) then both AeB and AoB have existential import with respect to both A and B. It follows immediately that all universal categorical statements have existential import with respect to both terms. If AaB and AeB is a fair representation of the use of statements in normal natural language of All A is B and No A is B respectively, then the following example consequences arise:

"All flying horses are mythological" is false if there are no flying horses.

If "No men are fire-eating rabbits" is true, then "There are fire-eating rabbits" is true and so on.

If it is ruled that no universal statement has existential import then the square of opposition fails in several respects (e.g. AaB does not entail AiB) and a number of syllogisms are no longer valid (e.g. BaC,AaB->AiC).

These problems and paradoxes arise in both natural language statements and statements in syllogism form because of ambiguity, in particular ambiguity with respect to All. If "Fred claims all his books were Pulitzer Prize winners", is Fred claiming that he wrote any books? If not, then is what he claims true? Suppose Jane says none of her friends are poor; is that true if she has no friends?

The first-order predicate calculus avoids such ambiguity by using formulae that carry no existential import with respect to universal statements. Existential claims must be explicitly stated. Thus, natural language statements—of the forms *All A is B, No A is B*, *Some A is B*, and *Some A is not B*—can be represented in first order predicate calculus in which any existential import with respect to terms A and/or B is either explicit or not made at all. Consequently, the four forms *AaB, AeB, AiB*, and *AoB* can be represented in first order predicate in every combination of existential import—so it can establish which construal, if any, preserves the square of opposition and the validity of the traditionally valid syllogism. Strawson claims such a construal is possible, but the results are such that, in his view, the answer to question (e) above is *no*.

On the other hand, in modern mathematical logic, however, statements containing words "all", "some" and "no", can be stated in terms of set theory. If the set of all A's is labeled as s(A) and the set of all B's as s(B), then:

- "All A is B" (AaB) is equivalent to "s(A) is a subset of s(B)", or $s(A) \subseteq s(B)$

- "No A is B" (AeB) is equivalent to "The intersection of s(A) and s(B) is empty", or
- "Some A is B" (AiB) is equivalent to "the intersection of s(A) and s(B) is not empty", or
- "Some A is not B" (AoB) is equivalent to "s(A) is not a subset of s(B)"

By definition, the empty set is a subset of all sets. From this it follows that, according to this mathematical convention, if there are no A's, then the statements "All A is B" and "No A is B" are always true whereas the statements "Some A is B" and "Some A is not B" are always false. This, however, implies that AaB does not entail AiB, and some of the syllogisms mentioned above are not valid when there are no A's.

Syllogistic fallacies

People often make mistakes when reasoning syllogistically.

For instance, from the premises some A are B, some B are C, people tend to come to a definitive conclusion that therefore some A are C. However, this does not follow according to the rules of classical logic. For instance, while some cats (A) are black things (B), and some black things (B) are televisions (C), it does not follow from the parameters that some cats (A) are televisions (C). This is because in the structure of the syllogism invoked (i.e. III-1) the middle term is not distributed in either the major premise or in the minor premise, a pattern called the "fallacy of the undistributed middle".

Determining the validity of a syllogism involves determining the distribution of each term in each statement, meaning whether all members of that term are accounted for.

In simple syllogistic patterns, the fallacies of invalid patterns are:

- Undistributed middle: Neither of the premises accounts for all members of the middle term, which consequently fails to link the major and minor term.
- Illicit treatment of the major term: The conclusion implicates all members of the major term (P – meaning the proposition is negative); however, the major premise does not account for them all (i.e., P is either an affirmative predicate or a particular subject there).
- Illicit treatment of the minor term: Same as above, but for the minor term (S – meaning the proposition is universal) and minor premise (where S is either a particular subject or an affirmative predicate).
- Exclusive premises: Both premises are negative, meaning no link is established between the major and minor terms.
- Affirmative conclusion from a negative premise: If either premise is negative, the conclusion must also be.
- Negative conclusion from affirmative premises: If both premises are affirmative, the conclusion must also be.
- Existential fallacy: This is a more controversial one. If both premises are universal, i.e. "All" or "No" statements, one school of thought says they do not imply the existence of any members of the terms. In this case, the conclusion cannot be existential; i.e. beginning with "Some". Another school of thought says that affirmative statements (universal or particular) do imply the subject's existence, but negatives do not. A third school of thought says that the *any* type of proposition may or may not involve the subject's existence, and though this may condition the conclusion, it does not affect the form of the syllogism

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss Axioms of Syllogism.

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2. Discuss Figures and Moods

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8.4 FIGURES AND MOODS

In the conclusion, S and P have fixed positions but this is not the case with M. There are four ways in which M can occupy two places. These four ways are called four figures, i.e., the position of M determines the figure of argument. These figures are as follows: -

	I	II	III	IV
Major Premise:	M-P	P-M	M-P	P-M
Minor Premise:	S-M	S-M	M-S	M-S
Conclusion:	S-P	S-P	S-P	S-P

From this scheme it is clear that neither P nor S determines the figure of syllogism. History has recorded that Aristotle accepted only first three figures. The origin of the fourth figure is disputed. While Quine said that Theophrastus, a student of Aristotle, invented the fourth figure, Stebbing said that it was Gallen who invented the fourth figure. This dispute is not very significant. But what Aristotle says on the first figure is significant. Aristotle regarded the first figure as most ‘scientific’. It is likely that by ‘scientific’ he meant ‘satisfactory’. One of the reasons, which Aristotle has adduced, is that both mathematics and physical sciences establish laws in the form of the first figure. Second reason is that reasoned conclusion or reasoned fact is generally found in the first figure. Aristotle believed that only universal affirmative conclusion can provide complete knowledge and universal affirmative conclusion is possible only in the first figure. Aristotle quotes the fundamental principle of syllogism. 'One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C'. This principle can be expressed in this form:

Minor: A inheres in B
Major: B inheres in C
Conclusion: A inheres in C

Evidently, this argument satisfies transitive relation. This is made clear with the help of this diagram:

Let us mention four examples, which correspond to four figures.

FIGURE I

	M P	
Major Premise:	All Artists are Poets.	AAP
	S M	
Minor Premise:	All Musicians are Artists.	MAA
Conclusion:	∴ All Musicians are Poets.	MAP
	S P	

FIGURE II

	P M	
Major Premise:	All saints are pious.	SAP
	S M	
Minor Premise:	No criminals are pious.	CEP
Conclusion:	∴ No criminals are saints.	CES
	S P	

FIGURE III

	M P	
Major Premise:	All great works are worthy of study.	GAW
	M S	
Minor Premise:	All great works are epics.	GAE
Conclusion:	∴ Some epics are worthy of study.	EIW
	S P	

FIGURE IV

	P M	
Major Premise:	No soldiers are traitors.	SET
	M S	
Minor Premise:	All traitors are sinners.	TAS
Conclusion:	∴ Some sinners are not soldiers.	SOS
	S P	

We have to consider figures in conjunction with moods. Mood is determined by quality and quantity propositions, which constitute syllogism. Since there are four kinds of categorical proposition and there are three places where they can be arranged in any manner, there are sixtyfour different combinations in any given figure. Since there are four figures, in all, two hundred and fifty six ways of arranging categorical propositions are possible. These are exactly what we mean by moods. However, out of two hundred and fifty-six, two hundred and forty-five moods can be shown to be invalid by applying the rules and corollaries. So we have only eleven moods. There is no figure in which all eleven moods are valid. In any given figure only six moods are valid. They are as follows:

I.	AAA,	AAI	AII	EAE	EAO	EIO
II.	AEE	AEO	EAE	EAO	EIO	AOO
III.	AAI	AII	IAI	EAO	EIO	OAO
IV.	AAI	IAI	AEE	AEO	EAO	EIO

In all these cases, first letter stands for major premise, second for minor and third for conclusion. Moods are boxed in two ways. Moods within thick boxes are called strengthened moods, and moods within thin boxes are called weakened moods. It is important to know the difference between these two. When two universal premises can yield only particular conclusion, then such moods are called strengthened moods.

Notes

On the other hand, if we deduce particular conclusion from two universal premises, when it is logically possible to deduce a universal conclusion, then such moods are called weakened moods. When we recall that from universal premises alone particular conclusion cannot be drawn, both strengthened and weakened moods become invalid. Thus, the number of valid moods reduces to fifteen. In this scheme, we notice that EIO is valid in all the figures. Though EIO is valid in all figures, it is one mood in one figure and some other in another figure. Likewise, AEE is valid in the second and the fourth figures. But it is one mood in the second figure and different mood in the fourth figure. In the thirteenth century, one logician by name Pope John XXI, invented a technique to reduce arguments from other figures to the first figure. This technique is known as mnemonic verses. Accordingly, each mood, excluding weakened moods, was given a special name:

I.	Fig:	AAA	BARBARA	III.	Fig:	AAI	DARAPTI
		EAE	CELARENT			IAI	DISAMIS
		AII	DARII			AII	DATISI
		EIO	FERIO			EAO	FELAPTON
						OAO	BOCARDO
						EIO	FERISON
II.	Fig:	EAE	CESARE	IV.	Fig:	AAI	BRAMANTIP
		AEE	CAMESTRES			AEE	CAMENES
		EIO	FESTINO			IAI	DIMARIS
		AOO	BAROCO			EAO	FESAPO
						EIO	FRESISON

Syllogism can be tested using rules and corollaries. These are also known as general rules. There is one more method of testing syllogism. Every figure is determined by special rules. These are called special rules because they apply only to particular figure. These special rules also depend directly upon the axioms of quantity and quality. Therefore special rules can be proved. While doing so we shall follow the method of reductio ad absurdum because, it is a simple method.

I. Special rules of the first figure: M – P

S – M

S – P

1. Minor must be affirmative:

Proof :

1. Let minor be negative.

2. Conclusion must be negative. (From B3 and 1)

3. Conclusion distributes P. (From 2)

4. Major should distribute P. (From A2 and 3)

5. Major must be negative. (From A2 and 4)

6. Negative minor implies negative major.

7. Two premises cannot be negative (B1)

8. Minor must be affirmative. q.e.d.

2. Major must be universal:

Proof:

1. Let Major be particular.

2. Major undistributes M. (From 1)

3. Minor should distribute M. (From A1 and 1)

4. Minor should be affirmative. (First special rule)

5. Minor has to undistributed M.

6. Major should distribute M. (From A1)

7. Major must be universal. q.e.d.

Using these two special rules, valid moods can be distinguished from invalid moods.

II. Special rules of the Second figure: P – M

S – M

S – P

1. Only one premise must be negative:

Proof:

1. Let both premises be affirmative.

2. M is undistributed in affirmative statements.

3. (1) and (2) together contradict A1.

4. One premise must be negative. q.e.d.

2. Major should be universal:

Proof:

1. Let Major be particular.

2. Major undistributes P. (from 1)

3. Conclusion must be universal. (From B3 and first special rule).

4. Conclusion distributes P.

5. (2) and (4) together contradict A2.

6. Major should distribute P.

7. Major must be universal.

III. Special rules of the Third figure: M – P

M – S

S – P

1. Minor must be affirmative.

2. Conclusion must be particular.

(The reader is advised to try to prove these two rules).

IV. Special rules of the Fourth figure: P – M

M – S

S – P

1. If Major is affirmative, then minor must be universal.

Proof:

1. Let minor be particular when major is affirmative.

2. Major undistributes M.

3. Minor also undistributes M. (From 1)

4. (2) and (3) together contradict A1.

5. Minor should distribute M.

6. Minor must be universal.

2. If any premise is negative, major must be universal.

Proof:

1. Let major be particular, when one premise is negative.

2. Negative premise yields negative conclusion. (B3)

3. Negative conclusion distributes P.

4. Major should distribute P. (From 3 and A2)

5. Major must be universal.

6. (1) and (5) contradict one another.

7 Major must be universal. q.e.d.

3. If minor is affirmative, then, conclusion must be particular.

Proof:

1. Let conclusion be universal with affirmative minor.

2. Universal conclusion distributes S.
3. Minor should distribute S. (From A2 and 2)
4. Affirmative minor undistributed S.
5. (3) and (4) contradict one another.
6. Conclusion should undistribute S.
7. Conclusion must be particular.

8.5 FALLACIES

There are three important fallacies associated with categorical syllogism. They are fallacies of undistributed middle, illicit major and illicit minor. One example for each fallacy with explanation will suffice.

P M

Major Premise: All inscriptions are contents of historical study. IAC

S M

Minor Premise: All ancient coins are contents of historical study. AAC
Conclusion: All ancient coins are inscriptions. AAI

Ans: This argument is in the second figure. According to one special rule of the second figure, only one premise must be negative. Since this rule is violated M is undistributed in both the premises. The argument commits the fallacy of undistributed middle.

While mentioning the rule violated we can also say that according to one axiom of quantity, M should be distributed at least once. When this rule is violated this fallacy is committed. M P Major Premise: All sailors are strong. SAS M S Minor Premise: All sailors are men. SAM S P Conclusion: All men are Strong. MAS Ans: This argument is in the third figure. According to one special rule of the third figure, the conclusion must be particular. Since this rule is violated, the argument commits the fallacy of illicit minor. [The reader is advised to identify the second type of explanation.] P M Major Premise: Some rich people are merchants. RIM M S Minor Premise: No merchants are educated. MEE Conclusion: Some educated persons are not rich. EOR Ans: This argument is in the fourth figure. According to one special rule of the fourth figure, when a premise is negative major must be universal. This rule is violated by the argument and it commits the fallacy of illicit major. [The reader is advised to identify the second type of explanation.] In any deductive argument certain elements are constant. In syllogism, for example, quality and quantity and position of terms determine the structure of the argument. Keeping the structure constant if any term is replaced by any other term, the end result remains the same. Therefore the student can construct as many examples as he or she wants. The method of identifying the fallacy remains the same, if the structure remains the same.

8.6 REDUCTION OF ARGUMENTS

Reducing arguments from other figures to the first figure is one of the techniques developed by Aristotle to test the validity of arguments. It is because Aristotle held that the first figure is the perfect one; all others are imperfect. After reduction, if the argument is valid in the first figure, then it means that the original argument in any other figure is valid. This technique is quite mechanical. So, we are only required to know what exactly is the method involved. We will learn this only by practice.

II Figure

II Figure CESARE		I Figure CELARENT
P E M	→ Conversion→	M E P
S A M		S A M
S E P		S E P

No politicians are poets.	→ Conversion →	No poets are politicians.
All girls are poets.		All girls are poets.
∴No girls are politicians.		∴No girls are politicians.

In CESARE 'S' after 'E' indicates simple conversion. It shows that 'E' (major premise) must undergo simple conversion.

II Figure

II Figure	I Figure
CAMESTRES	CELARENT
P A M	M E S
S E M → conversion M E S	P A M
∴S E P	∴P E S conversion→S E P

'S' and 'T' after 'E' shows that 'E' (minor premise) should undergo simple conversion and both premises be transposed. 'S' after second 'E' shows that this 'E' (conclusion) should undergo simple conversion. [The student is advised to construct argument for this and subsequent reductions.]

P E M	→ Conversion→	M E P
M A S	→ Conversion→	S I M
SOP		SOP

As usual 'S' stands for simple conversion of 'E' (Major Premise) and 'P' stands for conversion per accedens of 'A' (Minor premise). This process is similar to first and third moods of III figure.

FRESISON		FERIO
P E M	→ Conversion→	M E P
M I S	→ Conversion→	S I M
S O P		S O P

A close observation of the above reductions reveals that they are to be performed according to certain parameters. The moods in the first figure are Barbara, Celarent, Darii and Ferio. Their initial consonants are arbitrarily found. For other figures, the initial consonants indicate to which of the first, the figure is to be reduced. Accordingly, Fesapo in the 4th figure is to be reduced to Ferio. Other consonants occurring in second, third and fourth figures' mnemonics indicate the operation that must be performed on the proposition indicated by the preceding vowel in order to reduce the syllogism to a first-figure syllogism. Certain 'keys' are the following. 's' indicates simple conversion; 'p' indicates conversion per accidens (by limitation); 'm' indicates the interchanging of the premises; 'k' indicates obversion; 'c' refers to the process that the syllogism is to be reduced indirectly. Meaningless letters in mnemonic terms are 'r', 't', 'l', 'n', and noninitial 'b' and 'd'. From reduction technique one point becomes clear. Originally, there were twenty-four valid moods. Later weakened and strengthened moods were eliminated on the ground that particular proposition (existential quantifier) cannot be deduced from universal propositions (universal quantifier) alone, and the number was reduced to fifteen. Now after reduction to first figure the number came down to four. Strawson argues that reduction technique is superior to axiomatic technique to which he referred in the beginning of his work 'Introduction to Logical Theory'. He regards the moods as inference-patterns. He argues that the path of reduction should be an inverted pyramid. At one particular point of time Strawson maintains that in addition to equivalence relation, we require opposition relation also to effect reduction.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Fallacies?

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2. What is Reduction of Arguments?

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8.7 ANTILOGISM OR INCONSISTENT TRIAD

This technique was developed by one lady by name, Christine Ladd-Franklin (1847-1930). This technique applies only to fifteen moods. The method is very simple. Consider Venn's results for all propositions. Replace the conclusion by its contradiction. This arrangement constitutes antilogism. If the corresponding argument should be valid, then antilogism should conform to certain structure. It must possess two equations and one in equation. A term must be common to equations. It should be positive in one equation and negative in another. Remaining two terms appear in equation. Consider one example for a valid argument as syllogism. There are five techniques to test the validity of arguments. Conditions of validity differ from traditional analysis to modern analysis. There are three important fallacies in this category.

8.8 VENN DIAGRAM TECHNIQUE

The Venn Diagram technique is shown for typical as well as unusual syllogisms. The problem of existential import is introduced by means of thesediagrams.

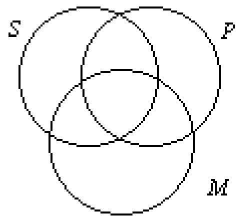
I. One good method to test quickly syllogisms is the Venn Diagram technique. This class assumes you are already familiar with diagramming categorical propositions. You might wish to review these now: Venn Diagrams.

A. A syllogism is a two premiss argument having three terms, each of which is used twice in the argument.

B. Each term (major, minor, and middle terms) can be represented by a circle.

C. Since a syllogism is **valid** if and only if the premisses entail the conclusion, diagramming the premisses will reveal the logical geography of the conclusion in a valid syllogism. If the syllogism is **invalid**, then diagramming the premisses is insufficient to show the conclusion must follow.

D. Since we have three classes, we expect to have three overlapping circles.



1. The area in the denoted *circle* represents where members of the class would be, and the area outside the circle represents all other individuals (the complementary class). The various area of the diagram are noted above.

2. *Shading* represents the knowledge that no individual exists in that area. *Empty space* represents the fact that no information is known about that area.

3. An "X" represents "at least one (individual)" and so corresponds with the word "some."

II. Some typical examples of syllogisms are shown here by their mood and figure.

A. **EAE-1**

1. The syllogism has an **E** statement for its major premiss, an **A** statement for its minor premiss, and an **E** statement for its conclusion. By convention the conclusion is labeled with **S** (the minor term) being the subject and **P** (the major term) being the predicate. The position of the middle term is the "left-hand wing."

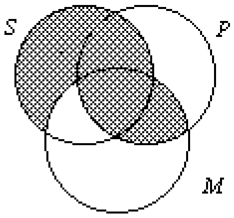
2. The form written out is

No **M** is **P**.

All **S** is **M**

No **S** is **P**.

3. Note, in the diagram below, how the area in common between S and P has been completely shaded out indicating that "No **S** is **P**." The conclusion has been reached from diagramming only the two premisses. All syllogisms of the form **EAE**-1 are valid.



B. **AAA**-1

1. This syllogism is composed entirely of "**A**" statements with the **M**-terms arranged in the "left-hand wing" as well.

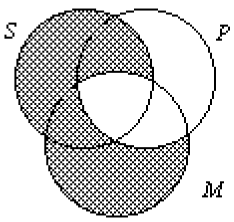
2. Its form is written out as

All **M** is **P**.

All **S** is **M**.

All **S** is **P**.

ote, in the diagram below, how the only unshaded area of **S** is in all classes. The important thing to notice is that this area of **S** is entirely in the **P** class. Hence, the **AAA**-1 syllogism is always valid. In ıary language the **AAA**-1 and the **EAE**-1 syllogisms are by far the frequently used.



C. **AII**-3

1. The **AII**-3 syllogism has the **M**-terms arranged in the subject position--the right side of the brick.

2. This syllogism sets up as

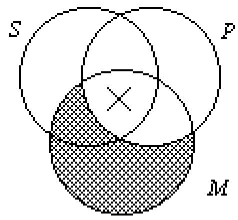
All **M** is **P**.

Some **M** is **S**.

Some **S** is **P**.

3. When diagramming the syllogism, notice how you are "forced" to put the "**X**" from the minor premiss in the area of the diagram shared by all three classes. The "**X**" cannot go on the **P**-line because the shading indicates this part of the SM area is empty. This "logical" forcing enables you to read-off the conclusion, "Some **S** is **P**."

4. This syllogism is a good example why *the universal premiss should be diagrammed before diagramming a particular premiss*. If we were to diagram the particular premiss first, the "**X**" would go on the line. Then, we would have to move it when we diagram the universal premiss because the universal premiss empties an area where the "**X**" could have been.



D. **AII**-2

1. The **AII**-2 has the **M** terms in the predicate of both premisses.

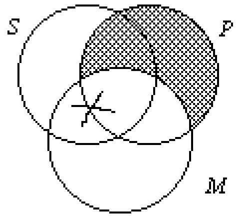
2. The syllogism is written out as

All **P** is **M**.

Some **S** is **M**.

Some **S** is **P**.

3. The diagram below shows that the "**X**" could be in the **SMP** area or in the **SPM** area. Since we do not know exactly which area it is in, we put the "**X**" on the line, as shown. *When an "**X**" is on a line, we do not know with certainty exactly where it is.* So, when we go to read the conclusion, we do not know where it is. Since the conclusion cannot be read with certainty, the **AII**-2 syllogism is invalid.



E. The final syllogism described here, the **EAO**-4 raises some interesting problems.

1. Notice that in this syllogism there are universal premisses with a particular conclusion.

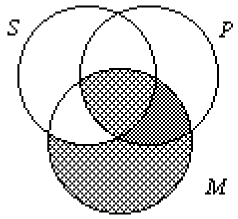
2. Its form is written out as

No **P** is **M**.

All **M** is **S**.

Some **S** is not **P**.

3. And its diagram is rather easily drawn as



4. When we try to read the conclusion, we see that there is no "**X**" in the **SMP** class. We must conclude that the syllogism is invalid because we cannot read-off "Some **S** is not **P**."

5. However, if we know that **M** exists, all the members of **M** have to be in the **SMP** class. These **M**'s are **S**'s as well. Hence, we know that some **S**'s are not **P**'s! In other words, the **EOA**-4 syllogism is valid if we know ahead of time the additional premiss "**M** exists."

6. Most contemporary logicians have concluded that we should not assume any class exists unless we have evidence.

a. We want to talk about theoretical entities without assuming their existence.

b. For example, in science and mathematics, our logic will apply when talking about circles, points, frictionless planes, and freely falling bodies even though these entities do not physically exist.

c. This diagram illustrates the contemporary topic called **the problem of existential import**.

Check Your Progress 3

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Antilogism or Inconsistent Triad?

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2. Discuss about Venn Diagram Technique.

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8.9 LET US SUM UP

As we know, our first example about roses was a categorical syllogism. Categorical syllogisms follow" If A is part of C, then B is part of C" logic.

Let's look at some more examples.

All cars have wheels. I drive a car. Therefore, my car has wheels.
Major Premise: All cars have wheels.
Minor Premise: I drive a car.
Conclusion: My car has wheels.

All insects frighten me. That is an insect. Therefore, I am frightened.

Major Premise: All insects frighten me.

Minor Premise: That is an insect.

Conclusion: I am frightened.

Conditional Syllogism

Conditional syllogisms follow an "If A is true, then B is true" pattern of logic. They're often referred to as hypothetical syllogisms because the arguments aren't always valid. Sometimes they're merely an accepted truth.

If Katie is smart, then her parents must be smart.

Major premise: Katie is smart.

Conclusion: Katie's parents are smart.

If Richard likes Germany, then he must drive an Audi.

Major premise: Richard likes Germany.

Conclusion: He must like all things German, including their cars.

Disjunctive Syllogism

Disjunctive syllogisms follow a "Since A is true, B must be false" premise. They don't state if a major or minor premise is correct. But it's understood that one of them is correct.

Major Premise: This cake is either red velvet or chocolate.

Minor Premise: It's not chocolate.

Conclusion: This cake is red velvet.

Major Premise: On the TV show Outlander, Claire's husband is either dead or alive.

Minor Premise: He's not alive.

Conclusion: Claire's husband is dead.

Enthymemes

An enthymeme is not one of the major types of syllogism but is what's known as rhetorical syllogism. These are often used in persuasive speeches and arguments.

Generally, the speaker will omit a major or minor premise, assuming it's already accepted by the audience.

He couldn't have stolen the jewelry. I know him.
Major Premise: He couldn't have stolen the jewelry.
Minor Premise: I know his character.
Her new purse can't be ugly. It's a Louis Vuitton.
Major Premise: Her new accessory can't be ugly.
Minor Premise: It's made by famous designer Louis Vuitton.
In an enthymeme, one premise remains implied. In the examples above, being familiar with someone or something implies an understanding of them.

Syllogistic Fallacy

Some syllogisms contain false presumptions. When you start assuming one of the major or minor premises to be true, even though they're not based in fact - as with disjunctive syllogisms and enthymemes - you run the risk of making a false presumption.

All crows are black. The bird in my cage is black. Therefore, this bird is a crow.
Major Premise: All crows are black.
Minor Premise: The bird in my cage is black.
Conclusion: This bird is a crow.

The scenery in Ireland is beautiful. I'm in Ireland. Therefore, the scenery must be beautiful.
Major Premise: The scenery in Ireland is beautiful.
Minor Premise: I'm in Ireland.
Conclusion: The scenery is beautiful.
Of course, not every black bird is a crow and not all of Ireland is beautiful. When preparing a speech or writing a paper, we must always make sure we're not making any sweeping generalizations that will cause people to make false presumptions.

Rules of Syllogism

There are six known rules of syllogism. However, they mainly apply to categorical syllogism, since that is the only category that requires three components: major premise, minor premise, conclusion. Here are six rules that will ensure you're making a strong and accurate argument.

Rule One: There must be three terms: the major premise, the minor premise, and the conclusion - no more, no less.

Rule Two: The minor premise must be distributed in at least one other premise.

Rule Three: Any terms distributed in the conclusion must be distributed in the relevant premise.

Rule Four: Do not use two negative premises.

Rule Five: If one of the two premises are negative, the conclusion must be negative.

Rule Six: From two universal premises, no conclusion may be drawn.

Further Examples of Syllogism

Literature

Syllogisms make for colorful literary devices. They explain situations indirectly, affording readers the opportunity to practice reasoning and deduction skills. Shakespeare was a master of many things, including syllogism. Here is an example of a syllogism fallacy in The Merchant of Venice:

Portia was a woman desired by many men. It was arranged she would marry the man who could correctly guess which of three caskets contained her portrait.

One casket was inscribed with, "Who chooseth me shall gain what many men desire." One man concluded that, since many men desired Portia, her portrait must be in that casket.

He was wrong. His assumption falls under the category of syllogistic fallacy. One cannot deduce that, since this casket contains what men desire, it's automatically the portrait. Men also desire fortune and power, for example. There wasn't enough evidence to leap from premise to conclusion here.

Philosophy

Socrates set up one of the most famous, and easily understand, examples of syllogism in philosophy. He clearly followed the rule of three components.

All men are mortal. Socrates is a man. Therefore, I am mortal.

This draws a clear picture of how one statement, when known to be universally true, should point perfectly to another clear claim, thus drawing an accurate conclusion.

Modern Culture

Keep syllogisms in mind when viewing advertisements. Many leaps are made in advertising, skipping either a major or minor premise:

Women love men who drive Lincoln MKZs.

Get ready for an enthymeme or syllogism fallacy. A blanket statement such as this skips one of the two required premises. Every time a woman likes a man, it can't be assumed he drives a Lincoln MKZ.

8.10 KEY WORDS

Paradox: A paradox is a statement or group of statements that leads to a contradiction or a situation which defies intuition or common experience.

8.11 QUESTIONS FOR REVIEW

1. Discuss Axioms of Syllogism
2. Discuss Figures and Moods
3. What is Fallacies?
4. What is Reduction of Arguments?

5. What is Antilogism or Inconsistent Triad?
6. Discuss about Venn Diagram Technique.

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8.13 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 8.2
2. See Section 8.3

Check Your Progress 2

1. See Section 8.4
2. See Section 8.5

Check Your Progress 3

1. See Section 8.6
2. See Section 8.7

UNIT 9: QUALIFICATION THEORY

STRUCTURE

9.0 OBJECTIVES

As we know inference is the main subject matter of logic. The term refers to the argument in which a proposition is arrived at and affirmed or denied on the basis of one or more other propositions accepted as the starting point of the process. To determine whether or not an inference is correct the logician examines the propositions that are the initial and end points of that argument and the relationships between them. This clearly denotes the significance of propositions in the study of logic. In this unit you are expected to study:

- the nature
- the definition
- the types and forms of propositions
- the difference between propositions and sentences and judgments

• the description of various types of propositions viewed from different standpoints like, composition, generality, relation, quantity, quality, and modality.

9.1 INTRODUCTION

Classical logic concerns itself with forms and classifications of propositions. We shall begin with the standard definition of proposition. A proposition is a declarative sentence which is either true or false but not both. Also a proposition cannot be neither true nor false. A proposition is always expressed with the help of a sentence. For example - the same proposition "It is raining" can be expressed in English, Hindi, and Sanskrit and so on. It means that two or more than two sentences may express the same proposition. This is possible only when proposition is taken as the meaning of the sentence which expresses it. Therefore sentence is only the vehicle of or the means of expressing a proposition. It is the unit of thought and logic whereas sentence is the unit of grammar. A sentence may be correct or incorrect; the grammatical rules determine this. A proposition may be true or false, the empirical facts determine the status.

The primary thing about a sentence is its grammatical form, but the primary thing about a proposition is its meaning and implication. The different types of sentences are not different types of propositions. Some types of sentences are not propositions at all. Sentences may be assertive, interrogative, and imperative. Only assertive types of sentences are propositions and rest of them are not (for more details, see below 9.3). A set of proposition make up an argument. Let us see what role propositions play and how logicians will be concerned in logic by taking a simple example of an argument:

All men are mortal. Proposition 1
All kings are men. Proposition 2
Therefore all kings are mortal. Proposition 3

Given these propositions as true or false, the logician will only find out whether the argument is valid or not by using certain rules that we shall learn later. Before we proceed further, it is of importance that we situate the discussion on “Proposition” in the whole context of the history of Logic itself.

As noted above, in Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject with the help of a copula. Aristotelian propositions take forms like "All men are mortal" and "Socrates is a man."

Propositions show up in modern formal logic as objects of a formal language. A formal language begins with different types of symbols. These types can include variables, operators, function symbols, predicate (or relation) symbols, quantifiers, and propositional constants. (Grouping symbols are often added for convenience in using the language but do not play a logical role.) Symbols are concatenated together according to recursive rules in order to construct strings to which truth-values will be assigned. The rules specify how the operators, function and predicate symbols, and quantifiers are to be concatenated with other strings. A proposition is then a string with a specific form. The form that a proposition takes depends on the type of logic.

The type of logic called propositional, sentential, or statement logic includes only operators and propositional constants as symbols in its language. The propositions in this language are propositional constants, which are considered atomic propositions, and composite propositions, which are composed by recursively applying operators to propositions. Application here is simply a short way of saying that the corresponding concatenation rule has been applied.

The types of logics called predicate, quantificational, or n-order logic include variables, operators, predicate and function symbols, and quantifiers as symbols in their languages. The propositions in these logics are more complex. First, terms must be defined. A term is

(i) a variable or

(ii) a function symbol applied to the number of terms required by the function symbol's arity. For example, if + is a binary function symbol and x, y, and z are variables, then x+(y+z) is a term, which might be written with the symbols in various orders. A proposition is

(i) a predicate symbol applied to the number of terms required by its arity,

(ii) an operator applied to the number of propositions required by its arity, or

(iii) a quantifier applied to a proposition. For example, if = is a binary predicate symbol and ∀ is a quantifier, then ∀x,y,z [(x = y) → (x+z = y+z)] is a proposition. This more complex structure of propositions allows these logics to make finer distinctions between inferences, i.e., to have greater expressive power.

In this context, propositions are also called sentences, statements, statement forms, formulas, and well-formed formulas, though these terms are usually not synonymous within a single text. This definition treats propositions as syntactic objects, as opposed to semantic or mental objects. That is, propositions in this sense are meaningless, formal, abstract objects. They are assigned meaning and truth-values by mappings called interpretations and valuations, respectively.

Propositions are called structured propositions if they have constituents, in some broad sense.

Assuming a structured view of propositions, we can distinguish between singular propositions (also Russellian propositions, named after Bertrand Russell) which are about a particular individual, general propositions, which are not about any particular individual, and particularized propositions, which are about a particular individual but do not contain that individual as a constituent.

By Aristotle

Aristotelian logic identifies a proposition as a sentence which affirms or denies a predicate of a subject with the help of a 'Copula'. An Aristotelian proposition may take the form "All men are mortal" or "Socrates is a man." In the first example the subject is "men", predicate is "mortal" and copula is "are". In the second example the subject is "Socrates", the predicate is "a man" and copula is "is".

By the logical positivists

Often propositions are related to closed formulae to distinguish them from what is expressed by an open formula. In this sense, propositions are "statements" that are truth-bearers. This conception of a proposition was supported by the philosophical school of logical positivism.

Some philosophers argue that some (or all) kinds of speech or actions besides the declarative ones also have propositional content. For example, yes–no questions present propositions, being inquiries into the truth value of them. On the other hand, some signs can be declarative assertions of propositions without forming a sentence nor even being linguistic, e.g. traffic signs convey definite meaning which is either true or false.

Propositions are also spoken of as the content of beliefs and similar intentional attitudes such as desires, preferences, and hopes. For example, "I desire that I have a new car," or "I wonder whether it will snow" (or, whether it is the case that "it will snow"). Desire, belief, and so on, are thus called propositional attitudes when they take this sort of content.

By Russell

Bertrand Russell held that propositions were structured entities with objects and properties as constituents. One important difference between Ludwig Wittgenstein's view (according to which a proposition is the set of possible worlds/states of affairs in which it is true) is that on the Russellian account, two propositions that are true in all the same states of affairs can still be differentiated. For instance, the proposition that two plus two equals four is distinct on a Russellian account from three plus three equals six. If propositions are sets of possible worlds, however, then all mathematical truths (and all other necessary truths) are the same set (the set of all possible worlds).

In relation to the mind, propositions are discussed primarily as they fit into propositional attitudes. Propositional attitudes are simply attitudes characteristic of folk psychology (belief, desire, etc.) that one can take toward a proposition (e.g. 'it is raining,' 'snow is white,' etc.). In English, propositions usually follow folk psychological attitudes by a "that clause" (e.g. "Jane believes that it is raining"). In philosophy of mind and psychology, mental states are often taken to primarily consist in propositional attitudes. The propositions are usually said to be the "mental content" of the attitude. For example, if Jane has a mental state of believing that it is raining, her mental content is the proposition 'it is raining.' Furthermore, since such mental states are about something (namely propositions), they are said to be intentional mental states. Philosophical debates surrounding propositions as they relate to propositional attitudes have also recently centered on whether they are internal or external to the agent or whether they are mind-dependent or mind-independent entities (see the entry on internalize and externalism in philosophy of mind).

9.2 HISTORY OF LOGIC AND PROPOSITION

Aristotle, the classical logician defines proposition as that which contains subject, predicate and a copula. “Rose is red” is a proposition. Here ‘Rose’ is the subject, ‘red’ is the predicate and ‘is’ is the copula. A subject is that about which something is said, a predicate is what is said about the subject and the copula is the link. Further, according to classical logicians copula should be expressed in the form of present tense only. That is why classical logicians talk of reduction of sentences into propositions. According to them all propositions are sentences but all sentences are not propositions. Subject-predicate logic ultimately gave rise to substance-attribute metaphysics in philosophy. Aristotle classifies proposition into four types.

They are as follows: Universal affirmative (A); Universal negative (E); Particular affirmative (I) and Particular negative (O). These propositions are called categorical or unconditional propositions because no condition is stated anywhere in the propositions. Letters within parentheses are standard symbols of respective propositions which are extensively used throughout our study of logic. “All men are mortal” is an example of ‘A’ proposition. “No men are immortal” is an instance of ‘E’ proposition. “Some men are intelligent” is an ‘I’ proposition and “Some men are honest” is an instance of ‘O’ proposition. Aristotle was the first thinker to devise a logical system. He holds that a proposition is a complex involving two terms, a subject and a predicate. The logical form of a proposition is determined by its quantity (universal or particular) and quality (affirmative or negative). The analysis of logical form, types of inference, etc. constitute the subject matter of logic. Aristotle may also be credited with the formulation of several metalogic propositions, most notably the Law of Noncontradiction, the Principle of the Excluded Middle, and the Law of Bivalence. These are important in his discussion of modal logic and tense logic. Aristotle referred to certain principles of propositional logic and to reasoning involving hypothetical propositions. He also formulated non-formal logical theories, techniques and strategies for devising arguments (in the Topics), and a theory of fallacies (in the Sophistical Refutations). Aristotle’s pupils Eudemus and Theophrastus modified and developed Aristotelian logic in several ways. The next major innovations in logic are due to the Stoic school. They developed an alternative account of the syllogism, and, in the course of so doing, elaborated a full propositional logic which complements Aristotelian logic. They also investigated various logical antinomies, including the Liar Paradox. The leading logician of this school was Chrysippus, credited with over a hundred works in logic. There were few developments in logic in the succeeding periods, other than a number of handbooks, summaries, translations, and commentaries, usually in a simplified and combined form. The more influential authors include Cicero, Porphyry, and Boethius in the later Roman Empire; the Byzantine scholiast Philoponous; and alFarabi, Avicenna, and Averroes in the Arab world. The next major logician of proposition is Peter Abelard, who worked in the early twelfth century. He composed an independent treatise on logic, the Dialectica, and wrote extensive commentaries. There are discussions of conversion, opposition, quantity, quality, tense logic, a reduction of de dicto to de re modality, and much else. Abelard also clearly formulates several semantic principles. Abelard is responsible for the clear formulation of a pair of relevant criteria for logical consequences. The failure of his criteria led later logicians to reject relevance implication and to endorse material implication. Spurred by Abelard's teachings and problems he proposed, and by further translations, other logicians began to grasp the details of Aristotle's texts. The result, coming to fruition in the middle of the thirteenth century, was the first phase of supposition theory, an elaborate doctrine about the reference of terms in various propositional contexts. Its development is preserved in handbooks by Peter of Spain, Lambert of Auxerre, and William of Sherwood. The theory of obligations, a part of non-formal logic, was also invented at this time. Other topics, such as the relation between time and modality, the conventionality of semantics, and the theory of truth, were investigated. The fourteenth century is the apex of mediæval logical theory, containng an explosion of creative work. Supposition theory is developed extensively in its second phase by logicians such as William of Ockham, Jean Buridan, Gregory of Rimini, and Albert of Saxony. Buridan also elaborates a full theory of consequences, a cross between entailments and inference rules. From explicit semantic principles, Buridan constructs a detailed and extensive investigation of syllogistic, and offers completeness proofs.

9.3 PROPOSITIONS AND SENTENCES

Propositions are stated using sentences. However, all sentences are not propositions. Let's look at a few examples of sentences:

1. Snakes are poisonous.
2. Some students are intelligent.
3. How old are you?
4. May God bless you!
5. What a car!
6. Vote for me.

The first two statements are assertions and we can say of these statements that they may either be true or false. Therefore they are propositions. However, we cannot say whether or not the question, 'How old are you?' is true or false. The answer to the question, 'I am 16 years old' may be true or false. The question is not a proposition, while the answer is a proposition. 'May God bless you' is a ceremonial statement and it is neither true nor false. Therefore, such statements are not propositions. 'What a car!' is exclamatory and has nothing to do with being true or false. Exclamatory statements are not propositions. 'Vote for me' is an appeal or command. We cannot attribute truth or falsity to it. Therefore, evocative statements are not propositions. We therefore need to distinguish between sentences and propositions. The differences are:

1. Propositions must be meaningful (meaningful in logical sense) sentences.

2. Propositions must have a subject, a predicate and a word joining the two, a sentence need not.

3. All propositions are either true or false, but sentences may or may not be.

4. Propositions are units of Logic, sentences are units of Grammar.

9.4 PROPOSITIONS AND JUDGMENTS

Till the nineteenth century, idealistic philosophers used the word, 'Judgment' instead of 'propositions'. Nowadays, a distinction is made between the two words. "Judgment" means 'pronouncing a formal decision'. "Proposition" means 'the result of judging'. Judgment is basically the attitude we take whereas proposition is that which we affirm or deny, accept or reject as true or false. Judgment is a mental act, a process, and an event in time. Proposition is time invariant. When we say 'All kings are mortal', it is a proposition. When we assert 'We believe that all kings are mortal', we are in fact taking an attitude, making a judgment. Sometimes, a statement may appear by itself to be a proposition. However, if one knows the context in which the statement is made, it may turn out that the proposition is really a judgment made.

Consider the statement: 'All foreigners are unacceptable'. By itself, it looks like a proposition, but what, if a speech is made and at the end the speaker concludes logically why 'all foreigners are unacceptable'. In such a case the speaker is actually passing a judgment. Sometimes, therefore, we need the context to distinguish a proposition from a judgment. It is only in the beginning of twentieth century that A.N. Whitehead and Bertrand Russell recognize varieties of propositions. According to them subject-predicate logic is only one form of propositions.

Notational Definition

So far, we have defined the meaning of the logical connectives by their introduction rules, which is the so-called verificationist approach. Another common way to define a logical connective is by a notational definition. A notational definition gives the meaning of the general form of a proposition in terms of another proposition whose meaning has already been defined. For example, we can define logical equivalence, written $A \equiv B$ as $(A \supset B) \wedge (B \supset A)$. This definition is justified, because we already understand implication and conjunction. As mentioned above, another common notational definition in intuitionistic logic is $\neg A = (A \supset$

⊥). Several other, more direct definitions of intuitionistic negation also exist, and we will see some of them later in the course. Perhaps the most intuitive one is to say that ¬A true if A false, but this requires the new judgment of falsehood. Notational definitions can be convenient, but they can be a bit cumbersome at times. We sometimes give a notational definition and then derive introduction and elimination rules for the connective. It should be understood that these rules, even if they may be called introduction or elimination rules, have a different status from those that define a connective.

Harmony

In the verifications definition of the logical connectives via their introduction rules we have briefly justified the elimination rules. In this section we study the balance between introduction and elimination rules more closely. In order to show that the two are in harmony we establish two properties: local soundness and local completeness. Local soundness shows that the elimination rules are not too strong: no matter how we apply elimination rules to the result of an introduction we cannot gain any new information. We demonstrate this by showing that we can find a more direct proof of the conclusion of elimination than one that first introduces and then eliminates the connective in question. This is witnessed by a local reduction of the given introduction and the subsequent elimination. Local completeness shows that the elimination rules are not too weak: there is always a way to apply elimination rules so that we can reconstitute a proof of the original proposition from the results by applying introduction rules. This is witnessed by a local expansion of an arbitrary given derivation into one that introduces the primary connective. Connectives whose introduction and elimination rules are in harmony in the sense that they are locally sound and complete are properly defined from the verificationist perspective. If not, the proposed connective should be viewed with suspicion. Another criterion we would like to apply uniformly is that both introduction and elimination rules do not refer to other propositional constants or connectives (besides the one we are trying to define), which could create

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a dangerous dependency of the various connectives on each other. As we present correct definitions we will occasionally also give some counterexamples to illustrate the consequences of violating the principles behind the patterns of valid inference.

Substitution Principle

We need the defining property for hypothetical judgments before we can discuss implication. Intuitively, we can always substitute a deduction of A true for any use of a hypothesis A true. In order to avoid ambiguity, we make sure assumptions are labelled and we substitute for all uses of an assumption with a given label. Note that we can only substitute for assumptions that are not discharged in the sub proof we are considering.

9.5 TYPES OF PROPOSITION

Propositions can be viewed from different standpoints and classified into different types:

STANDPOINT	TYPES OF PROPOSITIONS
Composition	Simple, Complex or Compound
Generality	Singular, General
Relation	Categorical, Conditional
Quantity	Universal, Particular
Quality	Affirmative, Negative
Modality	Necessary, Assertoric, Problematic
Significance	Verbal, Real

Composition - Simple Propositions

Examples: Love is happiness. Tiger is ferocious. All white men were dreaded by the red Indians. A simple proposition has only one subject and one predicate. Note that the subject 'All white men' is one subject though it has many words. Similarly 'Red Indians' is one predicate.

Composition – Complex or Composite Propositions

Examples: Violence does not pay and leads to unhappiness. She is graceful but cannot act. Either he is honest or dishonest. If John comes home, then you must cook chicken. 'She is graceful' is a simple proposition. 'Cannot act' can be written as 'She cannot act', which is a simple proposition again. These simple propositions are connected by a conjunction 'but'. When two or more simple propositions are combined into a single statement we get a complex or composite proposition.

Generality: Singular proposition

Examples: The dog wags its tail. George is my friend.
Kapil Dev is a good cricketer.

When in a proposition the subject refers to a definite, single object, the proposition is said to be singular proposition. A proper noun or a common noun preceded by a definite article 'the' forms the subject of such a proposition.

Generality - General Propositions

Examples: Children like chocolate. All hill stations are health resorts. Some people are funny. Few bikes come with fancy fittings. When in a proposition the subject refers to many objects, the proposition is said to be a general proposition. A common noun forms the subject of such propositions. When it is singular, the indefinite article 'a' is used. 'A dog' means any dog. It generalizes across all dogs. Words like 'some', 'few' refer to more than one object.

Relation - Categorical Propositions

Examples: The pillows are soft Junk food is not good for health Music is the food of love. A proposition that affirms or denies something without any condition is called a categorical proposition. Recall that a proposition has a subject, a predicate and a joining word. The joining word relates the two together. In the first example the subject, "the pillows" is joined to the predicate "soft" by the joining word "are'. In this proposition the softness of the pillow is asserted or affirmed. In the second example it is denied that junk food is good for health. Simple and general propositions are categorical in nature. In the above examples there are no conditions relating the subject and the predicate. Therefore they are called categorical propositions.

Relation: Conditional Propositions

Examples: If you study hard, then you will do well. Robert is either an athlete or a carpenter. A conditional proposition consists of two categorical propositions that are so related to each other that one imposes a condition that must be fulfilled if what the other asserts is to be acceptable. There are three types of conditional propositions:

1. Hypothetical proposition
2. Alternative proposition
3. Disjunctive proposition

1. Hypothetical Proposition

Examples: If (you are hungry), then (you can eat chocolates.) If (it doesn't rain), then (the harvest will be poor.) A hypothetical proposition consists of two categorical propositions. They are put within parentheses. The first part is called antecedent and the second part is called consequent. These two propositions are related in such a way that if the first is true then the second must be true if the second is false, then the first also is false.

However, if the first part is false, the second part may be true or may be false.

Example: If the sun shines then there is light --------------- ---------------
antecedent consequent

2. **Alternative Proposition**

Examples: John is either a professor or a musician either we play football or we play cricket John is either a doctor or the author of this book. An alternative proposition consists of two simple categorical proposition connected by 'either – or' and thus suggesting that any one of these two proposition may be true or both may be true. John may be a professor or may be a musician. It is also likely that John is both a professor and a musician. The two parts of an alternative proposition are known as alternant. Either alternant may be true or both may be true. The alternative proposition will be false only when both the alternant are false.

Either (Alternant)	***Or (Alternant)***	***Proposition***
John is a professor	John is a musician	
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

3. **Disjunctive Proposition**

Examples: It is not the case that both he is honest and he is dishonest. It is not the case that both the meat is boiled and roasted A disjunctive proposition consists of two simple categorical propositions (alternant) which are so related that both cannot be simultaneously true.

Note: The fact that both cannot be true at the same time is the only difference between an alternative and disjunctive proposition. Thus there may be examples which are common to both. In symbolic logic we use disjunctive for alternative and the third variety is called negation. Examples: Either he is in the class or he is in the playground.

Either (Alternant)	*Or (Alternant)*	*Proposition*
	John is in class	John is in playground
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE

Modality: Assertoric Proposition:

Examples: The earth moves round the sun. Objects far away appear small to the eyes. At zero degree centigrade water turns into ice. Eleven players form a cricket team. The earth is not perfectly round. When the claim or assertion made in a proposition is verifiable it is called an assertoric proposition. The assertion that the earth moves round the sun can be verified by scientific methods. If the result of such verification is true then the proposition is true.

Modality: Necessary Proposition:

Examples: Bachelors are unmarried male. The result of any number multiplied by zero is zero. A point has no dimension. Propositions which are always true by definition are called necessary propositions.

Modality: Problematic Proposition:

Examples: Perhaps he is a rich man. She may be happier off with him. There may be famine this year. In a problematic proposition we only guess the truth or falsity and make no definite assertion.

Quantity - Universal Proposition:

Examples: All boys in the team are educated. No politicians are honest. Shillong is a hill station.

When the predicate tells something about the entire class referred to by the subject term, it is called a universal proposition. The predicate term ‘educated’ refers to the entire class referred to by the subject term ‘all boys in the team’.

Quantity - Particular Proposition:

Examples: Some girls are beautiful. Some songs are classical. Some men are religious. When the predicate term tells something about an indefinite part of the class referred to by the subject term, it is called particular proposition.

Quality:

The early discussion on proposition from the standpoint of quantity was based on the subject class being quantified by the word all, some, no etc. When we discuss proposition from the standpoint of quality our focus will be on the ‘copula’ between the terms. A copula relates the two terms and is of some form of the verb ‘to be’- ‘is’, ‘are’, ‘is not’, ‘are not’ The copula either affirms or denies the relation between two terms

Quality: Affirmative Proposition

Examples: Some fruits are sweet. All computers are fast. Mr. John is bald. If the relation between the subject term and the predicate term is positive (or affirmative), the proposition is said to be affirmative. In this case the copula is of the form ‘is’ or ‘are’.

Quality: Negative Proposition:

Examples: Some fruits are not sweet. All computers are not fast. Mr. John is not bald. If the relation between the subject term and the predicate term is negative (or denied), the proposition is said to be negative. In this case the copula is of the form ‘is not’ or ‘are not’.

Categorical term

A categorical term is something that will be categorized, such as 'dog' and 'cat'. It is usually a collective statement such as 'all dogs' or 'some dogs'.

Categorical proposition

A categorical proposition is simply a statement about the relationship between categories. It states whether one category or categorical term is fully contained with another, is partially contained within another or is completely separate.

A dog is an animal

Some dogs are friendly

No dog is a cat

Propositions may have *quality*: either affirmative or negative.

They may also have *quantity*: such as 'a', 'some', 'most' or 'all'. The 'all' quantity is also described as being *universal* and other quantities *particular*.

Predicate and subject

The first term in the proposition is the subject. The second term is the predicate.

Some dogs (subject) *are friendly* (predicate)

Distribution

A categorical term is said to be *distributed* if the categorical proposition that contains it says something about *all* members of that categorical term. It is *undistributed* if the categorical proposition that contains it says does *not* something about *all* members of that categorical term.

Four types

There are four types of categorical proposition, each of which is given a vowel letter A, E, I and O. A way of remembering these is: Affirmative universal, nEgative universal, affIrmative particular and nOgative particular. To be more correct, A and I letters came from the

Latin *affirmo*, and E and O from the Latin *nego*.

Form	**Type**	**Quality**	**Quantity**	**Distribution of X**	**Distribution of Y**
All X is Y	A	Affirmative	Universal	Distributed	Undistributed
No X is Y	E	Negative	Universal	Distributed	Distributed
Some X is Y	I	Affirmative	Particular	Undistributed	Undistributed
Some X is not Y	O	Negative	Particular	Undistributed	Distributed

In this classification, 'some X is some Y' is I and 'some X is not some Y' is O, although it can be argued that these may be treated as an additional two variants.

Opposites

There are several types of opposition used in categorical propositions. These can be traditionally placed in the *Square of Opposition*.

A	<-- *Contraries* -->	E
^ | *Subalterns* | V	*Contradictories* (diagaonals)	^ | *Subalterns* | V
I	<-- *Subcontraries* -->	O

- Contraries cannot both be true, but both can be false.
- Subcontraries cannot both be false, but both can be true.
- Subaltern pairs can both be true or both be false.
- Contradictories cannot both be true and cannot both be false.

Opposites are also described in the *converse*, *obverse* and *contrapositive*.

Converse

The converse of a categorical proposition is categorical proposition where the predicate and subject of the original proposition are exchanged. Note that the quantity does not move with the subject or predicate.

No dogs are cats --> *No cats are dogs*

Some dogs are friendly creatures --> *Some friendly creatures are dogs*

All dogs are animals --> *All animals are dogs*

The converse of any true E or I proposition is also true (making it a useful test). A and O converses are seldom true.

Obverse

The obverse of a categorical proposition has predicate term replaced with its complement and quality of the proposition reverse.

All dogs are animals --> *No dogs are not animals*

No dogs are not dangerous --> *All dogs are dangerous*

The obverse of *all* types of true categorical proposition are also true.

Contrapositive

The contrapositive of a categorical proposition is formed by taking the complement of both subject and predicate and then reversing them.

All dogs are animals --> *All non-animals are not dogs*

Some dogs are friendly --> Some non-friendly creatures are not dogs

The contrapositive of any true A or O proposition is also true (making it a useful test). Contrapositives of E and I propositions are seldom true.

9.6 QUALITY AND QUANTITY

So far we have viewed a proposition from various standpoints like composition, relation, and modality and so on. More important of these are the standpoints of quality and quantity in viewing categorical propositions. Recall that:

Quantity: Universal Particular Quality: Affirmative Negative If we view a proposition from a combined stand point of quality and quantity, we get the following classification as in Aristotle's logic:

Quality	Classification		Forms of Proposition
1. Universal+ Affirmative		A	All (…) are/is (…)
2. Universal+ Negative		E	No (…) are/is (…)
3. Particular+ Affirmative		I	Some (…) are (…)
4. Particular+ Negative		O	Some (..) are not (..)

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) What is a proposition? Distinguish it from sentence.

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2) Mention Aristotelian classification of proposition.

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9.7 LET US SUM UP

In the above unit we have seen how important it is to reduce sentences to its logical form, namely propositions. However, while changing sentences to propositional forms the following points must be remembered.

1) The meaning of the original sentence must be faithfully preserved in the logical form too.

2) The proposition must express all its three parts in the proper order, viz. subject, copula and predicate.

3) The subject of the proposition can be found out by answering the question "Of what anything is being stated"

4) There must be a copula connecting subject and predicate.

5) When reducing a negative sentence to logical form. The sign of negation should go with the copula and with the predicate of the proposition.

6) Compound sentences must be split up in to simple sentences to construct propositions out of them.

7) The quantity of the propositions must be indicated clearly.

Attempts to provide a workable definition of proposition include

Two meaningful declarative sentences express the same proposition if and only if they mean the same thing.

thus defining proposition in terms of synonymity. For example, "Snow is white" (in English) and "Schnee ist weiß" (in German) are different sentences, but they say the same thing, so they express the same proposition.

Two meaningful declarative sentence-tokens express the same proposition if and only if they mean the same thing.

Unfortunately, the above definitions have the result that two identical sentences/sentence-tokens may appear to have the same meaning and thus express the same proposition and yet have different truth-values, e.g. "I am Spartacus" said by Spartacus and said by John Smith; and e.g. "It is Wednesday" said on a Wednesday and on a Thursday. These examples reflect the problem of ambiguity in common language resulting in mistaken equivocation of the statements. “I am Spartacus” spoken by Spartacus is the declaration that the individual speaking is called Spartacus and it is true. When spoken by John Smith it is a declaration about a different speaker and it is false. The term “I” means different things, so “I am Spartacus” means different things.

A related problem is when identical sentences have the same truth-value yet express different propositions. The sentence “I am a philosopher” could have been spoken by both Socrates and Plato. In both instances, the statement is true, but means something different.

These problems are addressed in predicate logic by using a variable for the problematic term, so that “X is a philosopher” can have Socrates or Plato substituted for X, illustrating that “Socrates is a philosopher” and

“Plato is a philosopher” are different propositions. Similarly, “I am Spartacus” becomes “X is Spartacus” where X is replaced with terms representing the individuals Spartacus and John Smith.

The example problems are therefore averted if sentences are formulated with sufficient precision that their terms have unambiguous meanings.

A number of philosophers and linguists claim that all definitions of a proposition are too vague to be useful. For them, it is just a misleading concept that should be removed from philosophy and semantics. W.V. Quine maintained that the indeterminacy of translation prevented any meaningful discussion of propositions, and that they should be discarded in favor of sentences. Strawson advocated the use of the term "statement".

9.8 KEY WORDS

Evocation: Evocation is the act of calling or summoning a spirit, demon, god or other supernatural agent, in the Western mystery tradition. Comparable practices exist in many religions and magical traditions.

Reduction: Reduction in philosophy is the process by which one object, property, concept, theory, etc., is shown to be entirely dispensable in favor of another.

Proposition: The term proposition has a broad use in contemporary analytic philosophy. The most basic meaning is a statement proposing an idea that can be true or false. It is used to refer to some or all of the following: the primary bearers of truth-value, the objects of belief and other "propositional attitudes" (i.e., what is believed, doubted, etc.), the referents of that-clauses, and the meanings of declarative sentences. Propositions are the sharable objects of attitudes and the primary bearers of truth and falsity. This stipulation rules out certain candidates for propositions, including thought- and utterance-tokens which are not sharable and concrete events or facts, which cannot be false.

9.9 QUESTIONS FOR REVIEW

1) What is a proposition? Distinguish it from sentence.
2) Mention Aristotelian classification of proposition.

9.10 SUGGESTED READINGS AND REFERENCES

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9.11 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. A proposition is the unit of thought and logic and carriers a definite truth-value. A proposition is expressed with the help of a sentence. While proposition is the unit of thought, sentence is the unit of grammar. The primary thing about the proposition is its logical form while for a sentence its primary thing is its grammatical form.

2. Aristotle has classified proposition into 4 kinds. They are as follows: 1 Universal affirmative (A Proposition) 2 Universal negative (E Proposition) 3 Particular affirmative (I Proposition) 4 Particular negative (O Proposition)

UNIT 10: QUANTIFICATION

STRUCTURE

10.0 OBJECTIVES

In this unit, we propose to introduce you:

- to a new set of rules to test the validity of arguments, which consist of general and singular propositions.
- to all the rules involved in testing the validity of arguments.
- to understand Aristotle's theory of syllogism against the background of symbolic logic.
- to the application of the new class of rules.

10.1 INTRODUCTION

Broadly speaking, there are two types of arguments: arguments consisting of statements, which are truth-functionally compound and arguments, which are neither truth-functional nor compound. This Chapter deals with the latter kind of arguments. Logic, which deals with this branch, is called predicate logic or quantification logic. It is a system of deductive logic that combines the analysis of terms with the analysis of statements by making use of the logical properties of quantifiers. Generally, this type of argument consists of two kinds of statements, called general and singular. All propositions accepted by traditional logic belong to these two categories. Both universal and particular propositions are called general because in these two kinds subject is general term, like men, horses, plants, etc. However, in a singular proposition, the subject refers to a definite individual. The individual may be a human being like 'Tendulkar' or an object like 'the farthest planet from the sun'. The difference between the truth functional statements on the one hand, and general or singular propositions on the other, is that none of the techniques discussed so far, helps us when arguments consisting of general or singular statements are analyzed. Since quantifying expressions are involved in such statements, quantification is another technique used in our mission to subject these propositions to close scrutiny.

Traditional logic or analysis of categorical proposition is the take-off point for quantification logic. Quantity of proposition and subject-predicate relation form the base. While subject of proposition stands for any individual, predicate stands for the attributes an individual may or may not possess. These individuals and attributes are denoted by lower case letters and upper case letters respectively. With regard to lowercase letters there is one restriction. Only letters from 'a' to 'w' are used to denote individuals. These are individual constants. Generally, the practice is to choose the first letter of the term to designate the individual. Therefore term like Tendulkar, Dhoni, etc, are represented as t, d, etc. While their attributes like cricketer, swimmer, politician, etc. are designated by C, S, P, etc., by using upper case letters. However, when 'politician' becomes subject of a proposition it is designated by 'p'. In logic, common noun may be subject or predicate. 'Tendulkar is a cricketer' is an example for common noun being used as attribute. Symbolically, it becomes 'Ct': it is a symbolized statement. First we write the symbol for attribute. This is followed by the symbol for subject. Such a statement can be true or false. When 'x' is used for individual constant, then we have propositional function, which is neither true nor false. For example, 'Bx' would be a statement like 'x is brave'. The process of obtaining propositions from propositional function is called 'instantiation'. Thus we can say, 'Chandran is brave' – it is a proposition we obtain from the propositional function 'Bx'. Accordingly, propositional function is an expression that contains one or more individual variables, such that when all its individual variables are replaced by individual constants the result is a symbolized statement. The symbol 'y' has a special role to play. It is used to denote an arbitrarily selected individual. In quantification, negation has the same symbol.

10.2 QUANTIFICATION: ITS MEANING

In mathematics and empirical science, quantification is the act of counting and measuring that maps human sense observations and experiences into quantities. Quantification in this sense is fundamental to the scientific method.

An important aspect of quantification is the substitution of instances. There are two ways in which substitution is being made. In the case of singular proposition, substitution of any individual constant ranging from 'a' to 'w' can be made to 'x' which is known as individual variable; this process is, as we have just seen, instantiation. Another method is through generalization. Accordingly, the process of quantification takes place when the given proposition is general. A general proposition is of two types; universal and particular. So we have two quantifiers denoting these two types. Quantifiers are symbols that are used to represent quantifying expression such as everyone / everything / all or someone / something. Thus there are universal or existential quantifiers. In symbols they are respectively as follows: '(x)' and '(∃x)'. Since they may be affirmative or negative, we have four kinds of propositions, which are represented as follows:

1. All Indians are mortal. (x) Mx

2. No Indians are mortal. $(x) \neg Mx$

3. Some Indians are moral. $(\exists x)\ Mx$

4. Some Indians are not mortal. $(\exists x) \neg Mx$

The symbols used on the right hand side need some explanation. The symbol (x) is expanded in several ways. It can read ‘for all values of x’ or ‘Given any x’ or simply ‘for every x’, etc., where ‘x’ stands for individual constant ‘Indians’ and ‘M’ stands for ‘mortal’. Therefore $\neg$ M x is read ‘x is not mortal’. The symbol ($\exists$x) is read ‘there exists at least one x such that ….’ ( ) is called universal quantifier and $\exists$ is called existential quantifier. If we substitute I (Indians) or P (Pakistanis) for x then we get a proposition, which may be true or false. Just as x is used as individual variable to denote the subject, two Greek letters ‘Φ’ (Phi) and ‘Ψ’ (Psi) are used to denote predicate. So they are called predicate variables. Using these variables, A, E, I and O propositions are represented as follows:

1. All Indians are mortal (A) $(x)\ \Phi x$
2. No Indians are mortal (E) $(x) \neg \Phi x$
3. Some Indians are mortal (I) $(\exists x)\ \Phi x$
4. Some Indians are not mortal (O) $(\exists x) \neg \Phi x$

Using class membership relation, general propositions are represented as follows:
1. $(x)\ \Phi x \equiv (x)\{x \epsilon \Phi => x \epsilon \Psi\}$ Where ϵ is read ‘element of’
2. $(x) \neg \Phi x \equiv (x)\{x \epsilon \Phi => x \epsilon \Psi\}$ Where ϵ is read ‘not an element of’
3. $(\exists x)\ \Phi x \equiv (\exists x)\{x \epsilon \Phi \Lambda x \epsilon \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \epsilon \Phi \Lambda x \epsilon \Psi\}$

Quantification is the act of giving a numerical value to a measurement of something, that is, to count the quanta of whatever one is measuring. Quantification produces a standardized form of measurement that allows statistical procedures and mathematical calculations. Quantitative research methods are based on a natural science, positivist model of hypothesis testing. In the social sciences these methods attempt to collect and analyze numerical data on social phenomena, seeking to understand the links between a relatively small number of attributes across a wide variety of cases. Thus, quantification is especially useful in describing and analyzing social phenomena on a larger scale.

THE RISE OF QUANTIFICATION

During the last centuries quantification has become immensely prevalent in the social sciences. Practices of quantification have been widely used in the West since the thirteenth century, and even before that. But only in the first part of the seventeenth century did the idea that social topics may be subjected to systematic quantitative analysis begin to acquire real dominance in Europe. These tendencies grew stronger during the nineteenth century, and by the first half of the twentieth century the "quantitative paradigm" had become extremely dominant in most of the social sciences, including economics, psychology, sociology, and political sciences. There are a few prominent explanations for this growing use of quantitative measures in western society and in the social sciences in particular. First, the growing prominence and success of the natural sciences, especially physics, drove social scientists to imitate their use of quantitative measures in the hope of acquiring similar success and precision (see for example the 2002 book How Economics Became a Mathematical Science, by Roy Weintraub). A second explanation emphasizes the rise of capitalism and the rational spirit in western societies, described by sociologist Max Weber in his 1905 book The Protestant Ethic and the Spirit of Capitalism. Weber describes a move toward a more rational, bureaucratic, and calculative life, and the increased tendency to quantify social entities and behaviors is well explained in light of these changes. Some scholars ascribe the proliferation of quantification mainly to the rise of the modern centralized state, in which public officials face the need to efficiently manage increasing populations and large-scale social institutions. Finally, in his 1995 book Trust in Numbers: The Pursuit of Objectivity in

Science and Public Life, Theodore Porter suggests another interesting explanation. Porter argues that the tendency toward quantification in modern society is not so much a response to the success of the natural sciences, as it is an attempt of weak professional groups to pacify social and political pressures for greater accountability. In other words, according to Porter, the surge of quantification in the social sciences was driven mainly by the desire to create an appearance of professionalism and gain legitimacy for social research and public policies.

THE MERITS OF QUANTIFICATION

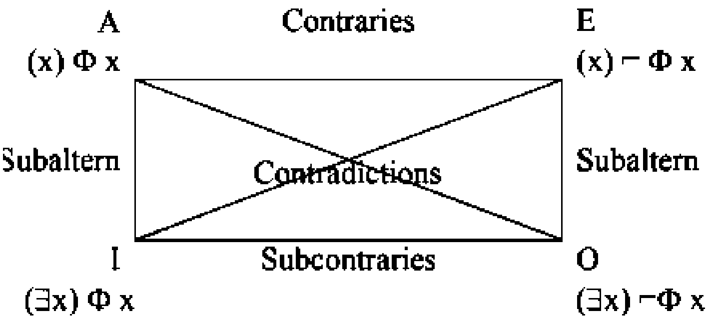
Quantification holds prominent advantages to scholars and policy makers. Its advocates believe that it increases precision and generalizability, while minimizing prejudice, favoritism, and nepotism in decision-making. According to this view, the decontextualized and valuefree mathematical symbols used in statistical analyses assist in achieving objectivity, stability, and fair judgment as decisions become more businesslike. In this sense the quantification and standardization of the social life have liberating and emancipatory effects. Quantification is also economical. Many feel that in today's world, with the inevitable avalanche of numbers that arises from the growing state apparatuses and with the fast advancing information revolution, there is simply too much information to be efficiently handled with detailed qualitative descriptions. Trying to make complicated decisions without finding a way to reduce the amount of information to be considered may be overwhelming. Quantification, therefore, serves as a necessary tool for organizing and discarding information, making the flux of data more manageable. It recognizes that people have bounded cognitive skills and can only process limited amounts of information. Quantification saves time, helps in making sense and analyzing large datasets, and facilitates large-scale research, planning, managing, and decision-making. In light of these advantages, some scholars believe that every aspect of the social world can, and in fact should be quantified. Psychologist Edward Thorndike, for example, claimed at the beginning of the twentieth century that “Anything that exists exists in a certain quantity and can be measured” (Custer 1996).

THE SHORTCOMINGS OF QUANTIFICATION

But many disagree with this approach. First, critics of quantification claim that it sacrifices the substance and authenticity of the information. Transforming social experiences into standardized numbers leads to alienation and distances many groups from these experiences. It also allows decision makers to escape accountability, as numbers and statistics become refuge from personal responsibility. In that sense, quantification is actually a way of making decisions without seeming to decide, as decisions are left to the numbers. Quantification, according to its opponents, symbolizes the takeover of the market economy over social life, eliminating values of recreation and spontaneity. Another problem is that quantification facilitates the emergence of new categories such as “the nation” or “public opinion.” These terms are actually materializations of complex social actions and institutions, but in the process of quantification they turn into “things.” In the process of quantification, important information is lost for the sake of simplicity and calculability. But in areas such as environmental preservation, intimate relationships, identities, rights, and religion, these attempts often distort the nature of the category and fundamental qualities disappear. At the same time, the dominance of quantification also erases existing objects and relations, making some social phenomena, which cannot be quantified, practically invisible. Finally, critics of quantification claim that it is often extended into areas in which it does not make statistical sense. This is especially true when measuring social entities, which are often flexible and subject to revision and change. For example social scientists often criticize the quantification of categories such as race and ethnicity, claiming that these are not real and stable entities, but rather fluctuating social definitions and classifications. This problem is exemplified in population censuses, in which some categories are invented and imposed on people by state officials, even when they do not coincide with personal identities and perceptions of self. In addition the interpretation of quantitative representations of social realities, such as race, fails to place these realities in the social context of the real world. This failure, in turn, may lead to misconceptions and erroneous judgments. Despite these problems, quantification is clearly a process that cannot be avoided. It is an important and viable component of today's social world, and there are few who would argue for returning to a prequantification world. Still, much more thought must be given to the problems of quantification and to its pitfalls. Researchers and policy makers must identify the places where it distorts the reality of social life and be much more cautious when applying it to social categories.

10.3 LOGICAL RELATIONS INVOLVING QUANTIFIERS

Our study begins with traditional square, which does not need any explanation. We know how A, E, I and O are denoted by quantification. Let us replace A, E, I and O by these quantifiers in the square:



With this background, we represent logical relations, viz., equivalence and contradiction as follows:

1. **Equivalence:**
   1) $(x)\,\Phi x \equiv \{\neg(\exists x)\neg\Phi x\}$
   2) $(x)\neg\Phi x \equiv \{\neg(\exists x)\,\Phi x\}$
   3) $(\exists x)\,\Phi x \equiv \{\neg(x)\neg\Phi x\}$
   4) $(\exists x)\neg\Phi x \equiv \{\neg(x)\,\Phi x\}$

2. Contradiction:

1)	$(x)\,\Phi x$	$(\exists x) \neg \Phi x$
2)	$(x) \neg \Phi x$	$(\exists x)\,\Phi x$
3)	$(\exists x)\,\Phi x$	$(x) \neg \Phi x$
4)	$(\exists x) \neg \Phi x$	$(x)\,\Phi x$

When we use predicate variable, the propositional forms are expressed as follows:

1)	$(x)\,\Phi x$	$\equiv$	$(x)\{\Phi x \Rightarrow \Psi x\}$
2)	$(x) \neg \Phi x$	$\equiv$	$(x)\{\Phi x \Rightarrow \neg \Psi x\}$
3)	$(\exists x)\,\Phi x$	$\equiv$	$(\exists x)\{\Phi x \wedge \Psi x\}$
4)	$(\exists x) \neg \Phi x$	$\equiv$	$(\exists x)\{\Phi x \wedge \neg \Psi x\}$

When we represent A, E, I & O with this new set, their equivalent forms also undergo changes.

1)	$(x)\{\Phi x \Rightarrow \Psi x\}$	$\equiv$	$\neg(\exists x)\{\Phi x \wedge \neg \Psi x\}$
2)	$(x)\{\Phi x \Rightarrow \neg \Psi x\}$	$\equiv$	$\neg(\exists x)\{\Phi x \wedge \Psi x\}$
3)	$(\exists x)\{\Phi x \wedge \Psi x\}$	$\equiv$	$\neg (x)\{\Phi x \Rightarrow \neg \Psi x\}$
4)	$(\exists x)\{\Phi x \wedge \neg \Psi x\}$	$\equiv$	$\neg (x)\{\Phi x \Rightarrow \Psi x\}$

If negations inserted behind the quantifiers on the RHS are removed, then automatically they become contradictions of respective propositions. A predicate like mortal is called simple predicate because the propositional function which, is used, has some true substitution instances and some false substitution instances. All substitutions to variable are called ‘substitution instances’. When simple predicates are negated, such formulas or statement forms ‘normal-form formula’.

10.4 QUANTIFICATION RULES

The rules of inference and replacement are augmented further with the addition of four more rules; universal instantiation (UI), universal generalization (UG), existential instantiation (EI) and existential generalization (EG). With the help of these rules and rules of inference and replacement any argument consisting of general or singular propositions or both can be tested. Before we apply these rules to test the validity of arguments, it is necessary that we know what these rules mean.

1. Universal Instantiation (UI): This rule says that any substitution instance of a propositional function can be validly deduced from a universal proposition. A propositional function always consists of variable ‘x’. Therefore any instance which is a substitution for ‘x’ must be a constant from ‘a’ through ‘w’. These letters signify subject in traditional sense, and in modern sense, an ‘instance of a form’. To transform such proposition ‘x’ is replaced by another Greek letter ‘ν’ (nu) when the function is universal quantifier, then ‘ν’ becomes universal instantiation.

$$\frac{(x)\,\Phi x}{\therefore \Phi \nu} \qquad \text{(where ‘}\nu\text{’ is any individual symbol)}$$

2. Universal Generalization (UG): This rule helps us to proceed to generalization after an arbitrary selection is made to substitute for ‘x’. In UG ‘arbitrary selection’ is very important because as the name itself suggests, generalization always proceeds from individual instance. And there is choice involved. In this sense, selection is ‘random’ or arbitrary. The letter ‘y’ is the symbol of ‘arbitrary’ selection. This process is called generalization because the conclusion is a universal proposition. If we recall the traditional rules of syllogism, universal conclusion follows from universal premise only. Therefore the process is from universal to universal through an individual. When ‘y’ replaces ’x’ there is generalization. When universal quantifier describes the proposition, it becomes.

$$\frac{\Phi y}{\therefore (x)(\Phi x)} \qquad \text{(where ‘y’ refers to any arbitrarily selected individual)}$$

3. Existential Instantiation (EI): This rule is applicable when the proposition has existential quantifier and any symbol ranging from a through w is used as a substitute for the individual variable x. We infer

the truth of any substitution instance from existential quantification. However, this rule has a clause. The constant, say 'a' which we use to substitute for x should not have occurred any where earlier in that context. It only means that in the same argument EI cannot be used twice when the substitution instance is only one.

$\underline{\Phi\ v}$
$\therefore (\exists x)\ \Phi x$ (where 'v' is any individual symbol)

4. Existential Generalization (EG): This rule states that from any true substitution instance of a propositional function, an existential quantification of that function can be validly deduced. In other words, an individual constant which appears in earlier steps, is replaced by x in the conclusion.

$\underline{(\exists x)\ \Phi x}$
$\therefore \Phi\ v$ (where 'v' is any individual constant other than y that has no prior occurrence in the context)

We should know why there is restriction on the use of EI. Suppose that 'a' is the constant whose existence is definite. We are not sure whether there is any other constant. In the earlier step 'a' is regarded as 'b'. The fact that 'a' is 'b' is not adequate enough to conclude that a is c in some other step when there is no reference of any to it in the premise. Since the logical constant a is used in existential mode, it is mandatory that EI should be used in the very first step of the proof. If it occupies any other position, then it is wrong.

10.5 TESTING THE VALIDITY OF SYLLOGISM

It is a matter of great interest to know that the rules of quantification project syllogism in a new perspective, which helps us to abandon the rule of distribution of terms, which is not only cumbersome in presentation but also time consuming. Further, quantification rules can be used to test non-syllogistic arguments also subject to the condition that only general and singular propositions find place in such arguments. Let us use the following arguments to illustrate these rules.

1\. 1). All Indians are Asians.

2). Tendulkar is an Indian.

3). ∴ Tendulkar is an Asian.

This is symbolized as follows: (x) (Ix=>Ax)

It

∴ At

The formal proof is constructed as follows:

1).	(x)(Ix => Az)		
2).	It	/∴ At	
3).	It => At	1,	U.I.
4).	∴ At	3, 2,	M.P.

In this particular argument only one premise is general. However, the argument may consist of only general propositions in which case slightly different procedure has to be followed. Consider this argument

2\. 1) All politicians are voters.

2) All ministers are politicians.

3) ∴ All ministers are voters.

When symbolized it becomes:

1) (x)(Px => Vx)

2) (x)(Mx => Px) /∴ (x)(Mx => Vx)

The formal proof is as follows:

1)	(x)(Px=>Vx)		
2)	(x)(Mx=>Px) /∴ (x){Mx=>Vx}		
3)	Pa => Va	1,	U.I.
4)	Ma => Pa	2,	U.I.
5)	Ma => Va	4, 3,	H.S.
6)	∴ (x)(Mx => Vx)	5,	U.G.

When the individual variable x is instantiated by any constant, then quantifier goes. We do not quantify individual or individuals. Now coming to the 6th step, it may be mentioned that if one substitution instance is true for a given structure then all substitution instances must be true for that structure. Further the universal quantification of a propositional function is true if and only if all substitution instances are true. (The 6th line is not a part of the proof) In the third and the fourth steps we have applied universal instantiation because two premises are universal and we have substituted the constants for variables.

UG can be applied in the following manner. Add the sixth line to the proof system after we replace x by y at all stages. Then we have the application of UG.

1)	(x){Px => Vx}		
2)	(x){Mx => Px}	/∴ (x){Mx => Vx}	
3)	Py => Vy	1,	U.I.
4)	My => Py	2,	U.I.
5)	My => Vy	3, 4,	H.S.
6)	∴ (x){Mx => Vx}	5	U.G.

These two examples suggest that while testing the validity of arguments UI has to be used necessarily though EI may not be necessary. The situation is similar to the traditional formation of rules of syllogism, which hint that without particular propositions it is possible to construct a valid argument, but not without universal propositions. Now consider an argument, which has a particular proposition. Since one proposition is particular, it is imperative that the conclusion must be particular.

3. 1) All politicians are Voters.
   2) Some ministers are politicians.
   ∴Some ministers are Voters.

By now the method of symbolization should be familiar.

1) (x){Px => Vx}

2) (∃x{Mx Λ Px} / (∃x{Mx Λ Vx}

3) Ma Λ Pa 2, E.I.

4) Pa => Va 1, U.I.

5) Pa Λ Ma 3, Com.

6) Pa 5, Simp.

7) Ma 5, Simp.

8) Va 4, 6, M.P.

9) Ma Λ Va 7, 8, Conj.

10) (∃x) (Mx Λ Vx) 9, I.G.

Let us examine why the restriction of EI must be honoured. Consider a fallacious argument.

1) Some animals are herbivorous.

2) Some animals are men. Some men are herbivorous.

When symbolized the argument becomes:

1) (∃x){Ax Λ Hx}

2) (∃x){Ax Λ Mx} / (∃x)(Mx Λ Hx)

3) Aa Λ Ha 1, E.I.

4) Aa Λ Ma 2, E.I. (Error)

4th Step is erroneous. The second premise tells us that there is at least one thing that is both an animal and herbivorous. It does not permit us to conclude that it should also be regarded as man. Therefore a second use of EI leads to error.

10.6 MULTIPLY GENERAL PROPOSITIONS

There are two types of general proposition; singly general and multiply general. If a general proposition has only one quantifier, then it is called singly general. Until now, we considered only propositions of former kind. If a general proposition consists of two or more than two quantifiers, then such a proposition is called multiply general propositions. Consider, for example, this proposition: “If all Indians play cricket, then there are at least some Asians who play cricket.” Its symbolization is as follows:

Notes

1) All Indians play cricket: (x){Ix => Px}

2)There are at least some Asians who play cricket:(∃x){Ax Λ Px} Now the symbolization of the whole sentence is as follows: {(x)(x => Px)} => {(∃x)(Ax Λ Px)} Depending upon the complexity of the given statement quantifiers may occur any number of times.

10.7 THE STRENGTHENED RULE OF C.P. AND QUANTIFICATION

In the previous unit, we learnt that assumption is different from conditional proof and that assumption does not include the conclusion, which depends solely on the premise. A few examples will illustrate how an argument can be tested using these techniques.

1. 1) (x)[Cx => Dx]

2) (x)[Ex => ⌐ Dx] (x)[Ex => ⌐ Cx]

The argument is written in standard form;

1)	(x)[Cx => Dx]		
2)	(x)[Ex => ⌐ Dx]	/ ∴ (x)[Ex => ⌐ Cx]	
→ 3)	Ey		
4)	Cy => Dy	1,	U.I.
5)	Ey => ⌐ Dy	2,	U.I.
6)	⌐Dy	5, 3,	M.P.
7)	⌐Cy	4, 6,	M.T.
8)	Ey => ⌐ Cy	3, 7,	C.P.
9)	(x)[Ex => ⌐ Cx]	9,	U.G.

From (1) two aspects become clear. The limit of assumption ends, when CP is used. So next step does not depend upon this assumption. Second, since we are making an assumption, in place of ‘x’ only ’y’; an arbitrary chosen symbol can be used. This explanation holds good whenever the strengthened rule of CP is used.

2.

1)	(x)[Nx => Ox]		
2)	(x)[Px => ¬ Ox]	/∴ (x) {(Nx Λ ¬Px) => Ox}	
→ 3)	Ny		
4)	Ny => Oy	1,	U.I.
5)	Py => ¬ Oy	2,	U.I.
6)	Oy	4, 3,	M.P.
7)	¬ Py	5, 6,	M.T.
8)	Ny Λ ¬ Py	3, 7,	Conj.
9)	(Ny Λ ¬ Px) => Oy	8, 6,	C.P.
10)	(x){ (Nx Λ ¬ Px) => Ox}		9, U.G.

10.8 PROVING INVALIDITY

The cardinal principle underlying the classification of arguments into good and bad is that true premises do not yield false conclusion. The easiest way of identifying the false conclusion in association with true premises is the method of assigning the truth-values to the components of statements. When the method of truth-values is extended to arguments with quantifiers one requirement has to be satisfied. We have to consider a nonempty model which is similar to a nonempty set. This model is the locus of our discussion. An argument involving quantifiers is valid if and only if to every nonempty model corresponds a logically equivalent and valid truth-functional argument. Similarly, an argument is invalid if there is a nonempty model to which corresponds a logically equivalent and invalid truth-functional argument. The crux of the matter is only this; an argument consisting of quantifiers is valid if and only if its truth-functional mode is valid and invalid if and only if its truth-functional mode is invalid. Since there is recourse to truth-functional mode, it is necessary to know how statements with quantifiers can be reduced to truth-functional compound statements. The very same truth-conditions which determine the truth-value of compound propositions also determine the truth-conditions of corresponding propositions with quantifiers. In the beginning of this section, we mentioned that an argument with quantifiers is valid if there is ‘at least’ one individual. It only means that there can be any number of individuals in a nonempty model. Suppose that there are only three men in the model of men, viz. a, b and c. In such a case the proposition ‘A’ can be represented in the following manner.

Notes

1. (x) (Φx) ≡ (Φa Λ Φb Λ Φc) The LHS is true if and only if Φa is true, Φb is true and Φc is true. If any one of them is false, then the LHS is false. Similarly, the proposition 'E' becomes

2. (x) (¬ Φx) ≡ (¬ Φa Λ¬ Φb Λ¬ Φc) If a, b and c are the only men in the model of men, then as in the previous case, in the present case also the LHS is true if and only if everyone of the three components is true. If any one of them is false then LHS also is false.

While the propositions with universal quantifiers are translated to the conjunction mode, those with existential quantifiers are reduced to the disjunction mode. If we persist with the same model, then

3. (∃x) (Φx) ≡ (Φa v Φb v Φc) 4. (∃x) (¬ Φx) ≡ (¬ Φa v ¬ Φb v ¬ Φc) From these four equations, it is clear that the truth status of propositions with quantifiers is determined by the truth-conditions of compound proposition. For example, consider (1). Even if one component on the RHS is false, then the LHS also turns out to be false. This is because conjunction is false when any component is false and in disjunction when any one component is true, the LHS is true. This type of relation is in perfect consonance with the definition of universal and existential quantifiers. Suppose that there is only one individual. Then two corollaries follow from this supposition, which are as follows. 1. (x) (Φx) ≡ Φa ≡ (∃x) (Φx) 2. Since there is only one true substitution instance (SI) to x, viz. a, we do not derive Φa from (x) (Φx) When there is only one individual any logical difference between universal and existential quantifiers also ceases to operate. Logically, there is a qualitative difference between a model containing only one individual and another model containing two or more than two individuals. (For the sake of convenience let us call the first model monadic and the second one polyadic model. If there are two individuals then the model is dyadic and if there are more than two then triadic and so on). There is a qualitative difference because in a monadic model an invalid argument may correspond to a valid truth-functional argument whereas the very same

argument in any other model may correspond to an invalid truth-functional argument. Let us consider an argument which is invalid from traditional angle.

1. All politicians are lawyers. All judges are lawyers.
   ∴ All judges are politicians.

   1. (x) [Px => Lx] 2. (x) [Jx => Lx] / ∴ (x) Jx => Px Since there is only one SI, this argument is logically equivalent to 3. p1:[Pa => La] 4. p2:[Ja => La] / ∴ Ja => Pa In a monadic model (x) (Φx) ≡ Φa ≡ (∃x) (Φx) ∴The argument is logically equivalent to 5. Pa Λ La 6. Ja Λ La / ∴ Ja Λ Pa If we assign the value 0 to any one of the components of the conclusion then not only the conclusion is false but also one of the premises becomes false. However, according to definition, the premises must be true. It is logically impossible to derive a false conclusion from true premises. Therefore in this case the argument is valid. However, the same argument is invalid in a dyadic model.

Before we consider an example for an argument in a dyadic model, let us consider the structure of the model. [(x) (Φx)] ≡ [Φa Λ Φb] [(x) ⌐ (Φx)] ≡ [⌐ Φa Λ⌐ Φb] (∃x) (Φx) ≡[Φa v Φb] (∃x) ⌐ (Φx) ≡[⌐ Φa v ⌐ Φb] Where a and b are two individuals who (or which) are the members of a dyadic model

2. Now let us symbolise the previous argument

1. p1: (x) [Px => Lx]

2. p2: (x) [Jx => Lx] / ∴ (x) Jx => Px Since we are considering a dyadic model the symbolic presentation is logically equivalent to

3. (Pa => La) Λ (Pb => Lb)

4. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb) Assign 0 to Pa and 1 to the rest. The result can be computed as follows

5. (Pa => La) Λ (Pb => Lb) 0 1 1 1 1 1 1 6. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb) 1 1 1 1 1 1 1 1 0 0 0 1 1 1

The conjunction of the truth-values which are boxed in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model. To become familiar with this method let us work with some more problems.

3. (x) (Dx => ⌐ Ex) (x) (Ex => Fx) / ∴(x) (Fx => ⌐ Dx) Let us restrict this argument to a dyadic model. If this argument is invalid in this model, then it is invalid in all other polyadic models. The logically equivalent form of 3 is as follows.

1. (Da => ⌐ Ea) Λ (Db => ⌐ Eb)

2. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db) Assign 0 to ⌐ Da. In accordance with the law of contradiction Da = 1. Similarly, ⌐ Db is assigned 0. Therefore Db = 1. Assign 1 to ⌐ Ea. Ea takes 0. Assign 1 to ⌐ Eb. Eb takes 0. Assign 1 to Fa and Fb. The result can be computed as follows. 3. (Da => ⌐ Ea) Λ (Db => ⌐ Eb) 1 1 1 1 1 1 1 4. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db) 0 1 1 1 0 1 1 1 0 0 0 1 0 0 In this argument also the conjunction of the truth-values boxed in 3and 4 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

4.

1. (∃x) (Jx Λ Kx)

2. (∃x) (Kx Λ Lx) / ∴(∃x) (Lx Λ Jx)

We shall consider this argument also in a dyadic model. This is logically equivalent to

3.(Ja Λ Ka) v (Jb Λ Kb)

4. (Ka Λ La) v (Kb Λ Lb) /∴(La Λ Ja) v (Lb Λ Jb)

There is a difference between this argument and the previous arguments. In this

argument the premises and conclusion are disjunctive unlike the previous arguments which have conjunctive statements. The difference is due to quantifiers. In case of universal quantifiers conjunction is the connective whereas in case of existential quantifiers disjunction is the connective.

Assign the truth-values as follows; 0 to La and Jb and 1 to the rest. The result is

computed as follows.

5.(Ja Λ Ka) v (Jb Λ Kb)

1 1 1 1 0 0 1

6. (Ka Λ La) v (Kb Λ Lb) /∴(La Λ Ja) v (Lb Λ Jb)

1 0 0 1 1 1 1 0 0 1 0 1 0 0

In this argument also the conjunction of the truth-values which are boxed in 5 and

6

yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

10.9 NON-SYLLOGISM

All arguments need not be syllogistic even though they consist of two premises and a conclusion. Relational argument is one such example.

1. Bangalore is to the west of Chennai. Mangalore is to the west of Bangalore. ∴Mangalore is to the west of Chennai. Aristotelian system does not regard this class of arguments as syllogistic though this can be shown to be valid in symbolic representation, but it results in the distortion of the meaning of statements. If we try to retain the meaning, then it becomes impossible to demonstrate the validity or invalidity, as the case may be. Apart from relational arguments, there is another class of arguments which consists of more than three terms and propositions. Consider this argument. Men (1) are both stupid (2) and dishonest (3). Some men are irritable (4). ∴ Some dishonest persons (3) are irritable (4). Terms are numbered so there is no confusion. However, the statements are misleading. If we regard a conjunctive proposition as one proposition, then in this argument there are three propositions. Even if the previous statement is accepted the argument cannot be syllogistic because there are four terms. If we give priority to simple propositions then the first premise has two simple propositions. Then we will have four propositions. Therefore this type of argument is classified as nonsyllogistic. To test this kind of argument we do not require any additional rule. Proper symbolization of this class of argument is important. The symbolization is as follows:

1. (x) [Mx => (Sx Λ Dx)]
2. (∃x) [Mx Λ Ix] / ∴ (∃x) (Ix Λ Sx). Its formal proof:

3.	[Ma Λ Ia]	2, E. I.
4.	Ma => (Sa Λ Da)	1, U. I.
5.	Ma	3, Simp.
6.	(Sa Λ Da)	4, 5, M. P.
7.	Sa	6, Simp.
8.	Ia	3, Simp.
9.	Ia Λ Sa	8, 7, Conj.
10.	(∃x) (Ix Λ Sx)	9, E.G.

The status of (1) calls for our attention. Had the first premise been regarded as a conjunctive proposition, then (1) ought to have been symbolized as 11. Sm Λ Dm It is a well known fact that conjunction does not have any equivalent form. Therefore (1) is not equivalent to (11). Consider another statement, which has a very different structure. Americans and Germans are pioneers in science. This statement actually means that a pioneer in science may be an American or a German. Surely, it does not mean at a pioneer in science is both an American and a German. Hence when this innocuous statement is translated into logical language, it becomes a disjunctive proposition with exclusive 'Or'. Nor is it a conjunctive proposition of the form Americans are pioneers in science and Germans are pioneers in science. This is so because a conjunctive proposition of this form means the same as saying that a pioneer in science is both an American and a German, which is absurd. Consider this argument: Americans and Germans are scientists. Some white men are Americans. Therefore, some white men are scientists. This argument is symbolized as follows:

1.	(x) [(Ax v Gx) => Sx]	
2.	(∃x) [Wx Λ Ax]	/ ∴ (∃x)[Wx Λ Sx]
3.	Wa Λ Aa	2, E.I.
4.	Aa	3, Simp.
5.	(Aa v Ga)	4, Add.
6.	(Aa v Ga) => Sa	2, U.I.
7.	Sa	6, 5, M.P.
8.	Wa	3, Simp.
9.	Wa Λ Sa	8, 7, Conj.
10.	(∃x)[Wx Λ Sx]	9, E.G.

In one particular sense, nonsyllogistic arguments are more significant than traditional syllogism for the simple reason that in any debate, whether based in science or politics, syllogism is seldom used. Application of nonsyllogistic arguments is widespread and more useful. Therefore there is greater need to become familiar with nonsyllogistic arguments.

Check Your Progress

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

I. Construct formal proofs of validity.

1.

1) (x)[Qx => Rx]

2) (∃x)[Qx v Rx] ∴ (∃x) Rx ---

2.

1) (x)[Sx => (Tx =>Ux)]

2) (x)[Ux=>(Vx ΛWx)] ∴ (x) [Sx => (Tx => Vx Λ Wx)] ------------------
----

3.

1) (x)[Dx => ⌐ Ex]

2) (x)[Fx => Ex] ∴ (x) [Fx => ⌐ Dx] --

4.

1) (∃x) [Jx Λ Kx]

2) (x) [Jx => Lx] ∴ (∃x) [Lx Λ Kx] --

10.10 LET US SUM UP

Some measure of the undisputed general importance of quantification in the natural sciences can be gleaned from the following comments:

"these are mere facts, but they are quantitative facts and the basis of science."

It seems to be held as universally true that "the foundation of quantification is measurement."

There is little doubt that "quantification provided a basis for the objectivity of science."

In ancient times, "musicians and artists ... rejected quantification, but merchants, by definition, quantified their affairs, in order to survive, made them visible on parchment and paper.

Any reasonable "comparison between Aristotle and Galileo shows clearly that there can be no unique lawfulness discovered without detailed quantification."

Even today, "universities use imperfect instruments called 'exams' to indirectly quantify something they call knowledge."

This meaning of quantification comes under the heading of pragmatics.

In some instances in the natural sciences a seemingly intangible concept may be quantified by creating a scale—for example, a pain scale in medical research, or a discomfort scale at the intersection of meteorology and human physiology such as the heat index measuring the combined perceived effect of heat and humidity, or the wind chill factor measuring the combined perceived effects of cold and wind.

Quantification is another set of rules, which augments the logical tools of test. It applies to arguments, which consist of general and singular propositions. Quantification rules must be used in conjunction with the rules of inference and replacement.

10.11 KEY WORDS

Dyadic: Dyadic is that which is composed of two sets of objects say A and B; if three sets or elements, then it is known as triadic; if four, then tartaric and if five, then pentomic.

Polyadic: Polyadic is that which comprises of many elements.

Quantification: In mathematics and empirical science, quantification is the act of counting and measuring that maps human sense observations and experiences into quantities. Quantification in this sense is fundamental to the scientific method.

10.12 QUESTIONS FOR REVIEW

1. Discuss the Proving Invalidity.
2. Discuss the Non-syllogism.

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10.14 ANSWERS TO CHECK YOUR PROGRESS

1) (x) [Qx => Rx]

2) (∃x) [Qx v Rx] /∴ (∃x) (Rx)

3) Qa v Ra 2, E.I.

4) Qa => Ra 1, U.I.

5) Ra 4, 3, M.P.

6) (∃x) Rx 5, E.G.

2

1) (x) [Sx => (Tx => Ux)]

2) (x) [Ux => (Vx Λ Wx)] /∴ (x) [Sx => {Tx => (Vx Λ Wx)}]

3) Sa => (Ta => Ua) 1, U.I.

4) Ua => (Va Λ Wa) 2, U.I.

5) (Sa Λ Ta) => Ua 3, Exp.

6) (Sa Λ Ta) => (Va Λ Wa) 5, 4, H.S.

7) Sa => (Ta => (Va Λ Wa)) 6, Exp.

8) ∴ (x) [Sx => {Tx => (Vx Λ Wx)}] 7, U.G.

3

1) (x) [Dx => ⌐ Ex]

2) (x) [Fx => Ex] /∴ (x) [Fx => ⌐ Dx]

3) Da => ⌐ Ea 1, U.I.

4) Fa => Ea 2, U.I.

5) Ea => ⌐ Da 3, Trans.

6) Fa => ⌐ Da 4, 5, H.S.

7) ∴ (x) [Fx => ⌐ Dx] 6, U.G.

4

1) (∃x) [Jx Λ Kx]

2) (x) [Jx => Lx] /∴ (∃x) [Lx Λ Kx]

3) Ja Λ Ka 1, E.I.

4) Ja => La 2, U.I.

5) Ja 3, Simp.

6) Ka 3, Simp.

7) La 4, 5, M.P.

8) La Λ Ka 7, 6, Conj.

9) (∃x) [Lx Λ Kx] 8, E.G.

UNIT 11: TECHNIQUES OF SYMBOLIZATION

STRUCTURE

11.0 OBJECTIVES

The main objective of this unit is to expose the techniques of symbolization and inadequacy of the rules of inference. While this is the primary objective, which is intended to be achieved, there is another objective. It is to demonstrate that logic is a growing science. If new techniques of testing arguments are invented, then logic stands on par with technological science where continuous inventions and improvements are the order of the day. This unit also serves to demonstrate a crucial factor that all arguments do not fall under one or two categories. Therefore the same set of rules cannot guarantee success. By the time you go through this unit, you should be in a position to identify the type of rules that are required to test given argument. This sort of ability can be acquired only by practice. Therefore, the arguments are designed in such a way that you are required to employ both the rules of inference and replacement.

11.1 INTRODUCTION

Not all arguments can be tested only with the rules of inference, though as shown in the previous unit, highly complex and diverse arguments succumb to these rules. Just as modern logic tried to supplement traditional logic, within modern logic, the need was felt to supplement the rules of inference. Hence we have the rules of replacement. The structure of argument may be such that it may require only the rules of replacement or only the rules of inference as we found it out in the previous unit or both. We have ten such rules, which are called the rules of replacement. The difference between these two sets of rules is that the rules of inference are themselves inferences whereas rules of replacement are not. However, the rules of replacement are restricted to change or change in the form of statements. For example, A or B is changed to B or A, or AΛ (B v C) is changed to (A Λ B) v (AΛC). Also, in the mode of application of rules there is a restriction. Any rule of inference should be applied to the whole line only, as mentioned in the previous unit, whereas any rule of replacement can be applied to any part of the line. Suppose, for example, that a line consists of the expression ‘A => (B Λ D)’. The consequent part cannot be simplified. The reason is simple. Suppose that either B or D is false, when A is false. Implication is still true. But we do not know whether B is true or D is true. On the other hand, no such restriction applies to any one of the ten rules, which are called replacement rules. All rules of replacement are logically equivalent expressions (i.e., these biconditionals (exposed with the symbol ‘≡’ / ‘<=>’ will be tautologies (true in all substitution instances) and so their replacement would be free from mistakes).

11.2 QUANTIFICATION RULES: THE RULES OF REPLACEMENT

Rules of Replacement in Symbolic Logic: Formal Proof of Validity

In my previous post titled “Rules of Inference in Symbolic Logic: Formal Proof of Validity”, I discussed the way in which arguments are proven valid using the 10 rules of inference. In this post, I will discuss the 10 rules of replacement as another method that can be used to justify steps in the formal proof of validity.

Rules of replacement are logical equivalences or logically equivalent sentence forms, which allow us to replace or substitute one member of a pair in the process of proving the validity of arguments.

But why the rules of replacement?

They are important because there are cases wherein the 10 rules of inference may not be employed in proving or demonstrating the validity of arguments. Hence, when cases like this occur, the rules of replacement may be the best, if not the only, method that can be employed in proving the validity of arguments.

Rules of Replacement vs **Rules of Inference**

It might be worthwhile at this point to briefly sketch the major differences between rules of replacement and rules of inference before we proceed to discuss in great detail the nature and dynamics of the 10 rules of replacement.

For one, the rules of inference are forms of valid arguments, while the rules of replacement are forms of equivalent propositions. This is the reason why we have the symbol $\therefore$ (read as "therefore") in rules of inference, while in rules of replacement, as I will show later, we have the equivalent sign $\equiv$ (read as "if and only if") between two parts or propositions.

Rules of Replacement

There are 10 rules of replacement, namely: 1) Double Negation (D.N.), 2) Commutation (Comm.), 3) Association (Assoc.), 4) De Morgan's Theorem (D.M.), 5) Material Implication (M.I.), 6) Transposition (Trans.), 7) Distribution (Dist.), 8) Material Equivalence (M.E.), 9) Tautology (Taut.), and 10) Exportation (Exp.).

1. Double Negation

The form of double negation is as follows:

p ≡ ~ ~ p

In double negation, we can replace **~ ~ p** with **p** and vice versa because **~ ~ p** is absolutely the same with **p**. Hence, **~ ~ p** is equivalent to **p**. Consider the proposition below.

1. It is not true that Melbert is not studying. (p)

As we can see, the proposition above is a simple proposition (because there is no other component proposition) with two negation signs “not”. Thus, if we symbolize the proposition, then we have to symbolize the negation signs accordingly. So, if we let **p** stand for “It is not true that Melbert is not studying”, then the proposition is symbolized as follows:

~ ~ p

But if we analyze the proposition, it is clear that it means “Melber is studying” because it is not true that he is not studying. Hence, the proposition can also be symbolized as follows:

p

Therefore, **~ ~ p** is equivalent to **p**.

2. Commutation

The forms of a commutation are as follows:

(p • q) ≡ (q • p)

(p v q) ≡ (q v p)

The idea behind the law of commutation is that the order in which conjunctions and disjunctions are written is irrelevant in their truth value, unlike in a conditional proposition, for example, where the order of the proposition matters when it comes to its truth value. For example, if we have the proposition p ⊃ q, we cannot say that it’s the same with q ⊃ p. This is exactly what I meant above when I said that the rules of inference only have one direction. For sure, if we have the proposition “If it rains today, then the road is wet” (p ⊃ q), we cannot say that it is the same with “If the road is wet, then it rains today” (q ⊃ p). Of course, there are several factors that cause the wetness of the road other than the fact that it rains during the day. For instance, it might be true that a fire truck passes by and spills water on the road.

3. Association

The forms of association are as follows:

[(p • q) • r] ≡ [p • (q • r)]
[(p v q) v r] ≡ [p v (q v r)]

The underlying principle behind the law of association is that the grouping of conjunctions and disjunctions is irrelevant to their truth value. Just as in commutation, the grouping does not hold for conditional propositions. I need not explain further the fact that if we regroup conditional propositions, the thought will change; whereas in conjunctions and disjunctions, regrouping will not change the thought of the propositions.

4. De Morgan's Theorem (D.M.)

The forms of De Morgan Theorem are as follows:
~ (p • q) ≡ (~ p v ~ q)
~ (p v q) ≡ (~ p • ~ q)

De Morgan's Theorem allows substitution of disjunction for conjunction and vice versa. The first form above allows the replacement of a negated conjunction for a disjunction in which the quality is changed from positive to negative. The second form allows the replacement of a negated disjunction for a conjunction in which the quality is changed from positive to negative. Of course, since these propositions are equivalent, the replacement can be carried out in the opposite direction.

Just as in rules of inference, there are also variations in rules of replacement in terms of quality of propositions (that is, on negation and affirmation). For example, if we negate the **p** in the proposition **~ (p • q)** so that it would look like this **~ (~p • q)**, then its equivalent will also

Notes

change, which will look this (**p v ~ q**). Hence, for example, in De Morgan's Theorem, the variations will look like the following:

~ (~ **p** • **q**) ≡ (**p v ~ q**)
~ (**p** • ~ **q**) ≡ (~ **p v q**)
~ (~ **p** • ~ **q**) ≡ (**p v q**)
~ (~ **p v q**) ≡ (**p** • ~ **q**)
~ (**p v ~ q**) ≡ (~ **p** • **q**)
~ (~ **p v ~ q**) ≡ (**p** • **q**)

Please note that the same principle in terms of variations goes to the rest of the rules of replacement. I need not explain them again below. I believe the discussion above is enough to show to my readers that a change in the quality of one pair will result in a change of the quality of the other pair.

5. Material Implication

The form of a material implication is as follows:

(**p ⊃ q**) ≡ (~ **p v q**)

Material implication permits conditional and disjunctive propositions to be substituted for one another. For instance, if we have the proposition "If it rains today, then the road is wet", (**p ⊃ q**), then it can be replaced by "Either it does not rain today or the road is wet"(~ **p v q**).

6. Transposition

The form of a transposition is as follows:

(**p ⊃ q**) ≡ (~ **q ⊃ ~ p**)

Transposition permits the antecedents and consequents of conditional propositions to be interchanged but changing their quality at the same time. So that if the antecedents and consequents of the original propositions are positive, when interchanged they become negative.

For instance, if the original proposition is "If it rains today, then the road is wet" (**p ⊃ q**), then when interchanged using the rule of transposition the proposition will now read "If the road is not wet, then it does not rain today", (~ **q** ⊃ ~ **p**).

7. Distribution

The forms of distribution are as follows:

[**p** • (**q v r**)] ≡ [(**p** • **q**) v (**p** • **r**)]
[**p v** (**q** • **r**)] ≡ [(**p v q**) • (**p v r**)]

In logic, just as in mathematics, it is possible for us to distribute propositions across a parenthesis as long as the resultant proposition is equivalent to the original proposition.

For instance, the proposition "John is singing, while either Philippe is sleeping or Mary is studying", **p** • (**q v r**), can be replaced by "Either John is singing and Philippe is sleeping or John is singing and Mary is studying", (**p** • **q**) v (**p** • **r**).

8. Material Equivalence

The forms of a material equivalence are as follows:

(**p** ≡ **q**) ≡ [(**p** ⊃ **q**) • (**q** ⊃ **p**)]
(**p** ≡ **q**) ≡ [(**p** • **q**) **v** (~ **p** • ~**q**)]

The first form of material equivalence says that a biconditional proposition is equivalent to the joint assertion of two conditional propositions. For example, the biconditional proposition "The teacher will be absent if and only if he gets sick", (**p** ≡ **q**), can be replaced by the following proposition, "If the teacher will be absent, then he gets sick; and if the teacher gets sick, then he will be absent", [(**p** ⊃ **q**) • (**q** ⊃ **p**)]. The same principle goes to the second form of a material equivalence.

9. Tautology

The forms of a tautology are as follows:

p ≡ (**p v p**)
p ≡ (**p** • **p**)

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In tautology, the disjunction or conjunction of the same proposition is always equivalent to either one of the pairs. I need not explain further the forms of tautology. It's too obvious to do so.

10. Exportation

The form of exportation is as follows:

$[(\mathbf{p} \bullet \mathbf{q}) \supset \mathbf{r})] \equiv [\mathbf{p} \supset (\mathbf{q} \supset \mathbf{r})]$

The form of exportation says that the propositions $[(\mathbf{p} \bullet \mathbf{q}) \supset \mathbf{r})$ and $\mathbf{p} \supset (\mathbf{q} \supset \mathbf{r})$ both assert that **p** and **q** are antecedents of **r**.

we will construct a series of propositions based on the given argument using the rules of replacement. The goal here is to come up with a proposition that matches with the conclusion of the given argument. In other words, we will extract from the premises the conclusion of the given argument. Once we have successfully done this, then we can say that we have proven the validity of the argument.

In what follows, I will provide examples of arguments that have been proven valid already, and then do side but brief discussion on the process. It must be noted that proving the validity of arguments in symbolic logic cannot be done by applying the rules of replacement alone. Thus, in this relatively complicated arguments, we prove their validity by employing the 10 rules of inference and 10 rules of replacement. Of course, the 20 rules (10 rules of inference and 10 rules of replacement) will not be applied at once. We just need to choose which of the 20 rules are applicable in a particular situation.

Example 1

1) (p v q) ⊃ ~ ~ s
2) q v p
3) s ⊃ [r v (q v w)] /∴ w v (r v q)
4) p v q — 2 Comm.
5) ~ ~ s — 1, 4 M.P.
6) s — 5 D.N.
7) r v (q v w) — 3, 6 M.P.
8) (r v q) v w — 7 Assoc.
9) w v (r v q) — 8 Comm.

Solution:

First, we applied the rule in Commutation in Premise #2. See illustration below.

2) q v p
4) p v q — 2 Comm.

And then we applied Modus Ponens in Premise #1 and Premise #4. See illustration below.

1) (p v q) ⊃ ~ ~ s
4) p v q
5) ~ ~ s — 1, 4 M.P.

Next is we applied the rule in Double Negation in Premise #5. See illustration below.

5) ~ ~ s
6) s — 5 D.N.

Next is we applied Modus Ponens in Premise #3 and Premise #6. See illustration below.

3) s ⊃ [r v (q v w)]
6) s
7) r v (q v w) — 3, 5 M.P.

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And then we applied the rule in Association with Premise #7. See illustration below.

7) r v (q v w)
8) (r v q) v w   7 Assoc.

And finally, we applied the rule in Commutation in Premise #8. See illustration below.

8) (r v q) v w
9) **w v (r v q)**   8 Comm.

As we can see, Premise #9, which is **w v (r v q)**, matches with the conclusion of the given argument, which is, **w v (r v q)**. Hence, we have now proven the validity of the given argument.

Example 2

1) (T v P) ⊃ W
2) ~ W v (S • I)
3) ~ (~ S v ~ I) ⊃ ~ E
4) ~ V ⊃ E
5) ~ V /∴ **~ (T v P)**
6) E   4, 5 M.P.
7) ~ ~ E   6 D.N.
8) ~ ~ (~ S v ~ I)   3, 7 M.T.
9) ~ S v ~ I   8 D.N.
10) ~ (S • I)   9 D.M
11) ~ W   2, 10 D.S.
12) **~ (T v P)**   1, 11 M.T.

Example 3

1) ~ p v q
2) ~ r ⊃ ~ q
3) (p ⊃ r) ⊃ (~ s ⊃ t)
4) ~ s /∴ **t**
5) p ⊃ q   1 M.I.
6) q ⊃ r   2 Trans.
7) p ⊃ r   5, 6 H.S.
8) ~ s ⊃ t   3, 7 M.P.
9) **t**   4, 8 M.P.

Example 4

1) p • (q v r)
2) ~ p v ~ r /∴ **p • q**
3) (p • q) v (p • r)   1 Dist.
4) ~ p • r   2 D.M.
5) **p • q**   3, 4 D.S.

Example 5

1) (~ r v ~ s) ⊃ t
2) (p • ~ t) • s /∴ **r ≡ s**
3) p • (~ t • s)   2 Assoc.
4) ~ t • s   3 Simp.
5) ~ t   4 Simp.
6) ~ (~ r v ~ s)   1, 5 M.T.
7) r • s   6 D.M.
8) (r • s) v (~ r •~ s)   7 Add.
9) **r ≡ s**   8 M.E.

Example 6

1) (~ p v ~ s) v r /∴ **p ⊃ (s ⊃ r)**
2) ~ (p • s) v r   1 D.M.
3) (p • s) ⊃ r   2 M.I.
4) **p ⊃ (s ⊃ r)**   4 Exp.

1 De Morgan’s Law (De M.) ⌐ (p Λ q) ≡ (⌐ p v ⌐q)

⌐ (p v q) ≡ (⌐ p Λ ⌐ q)

2 Commutation Law (Com.) p v q ≡ q v p

p Λ q ≡ q Λ p

3 Double Negation(D.N.) ¬ (¬ p) ≡ p

4 Transposition (Trans.) (p => q) ≡ (¬ q => ¬ p)

5 Material Implication (Impl.) (p => q) ≡ ¬ p v q

6 Material Equivalence (Equiv.) (p ≡ q) ≡ {(p => q) Λ (q => p)}

(p ≡ q) ≡ {(p Λ q) v (¬ p Λ ¬q)}

7 Exportation (Exp.) {(p Λ q) => r} ≡ {p => (q => r)}

8 Tautology (Taut.) p ≡ p v p

p ≡ p Λ p

9 Association (Ass.) {p v (q v r)} ≡ {(p v q) v r}

{p Λ (q Λ r)} ≡ {p Λ q) Λ r}

10 Distribution (Dist.) {p Λ (q v r)} ≡ {(p Λ q) v (p Λ r)}

{p v (q Λ r)} ≡ {p v q) Λ (p v r)}

Some of these rules are structurally similar to some forms of immediate inference. For example, commutation law is similar, structurally, to simple conversion. Double negation is obversion. Transposition is what is called contraposition of hypothetical proposition in traditional logic. Finally, de Morgan's law is contradiction applied to disjunctive and conjunctive propositions. Now our task is well defined. We examine, initially, arguments which require only these rules.

a. TESTING THE VALIDITY OF ARGUMENTS

(THE RULES OF INFERENCE AND REPLACEMENT)

1 p Λ q

¬ (¬q v ¬p)

Ans:

1 p Λ q / ¬ (¬q v ¬p)

2 q Λ p 1, Com.

3 ¬ (¬q v ¬p) 2, (De.M.)

2 1 p => q / ¬q => ¬p

¬q => ¬p 1, Trans.

Notes

3 1 ⌐ p => q / ⌐q => p

2 ⌐q => ⌐ (⌐p) 1, Trans.

3. ⌐q => p 2, D.N.

Let us use symbols for propositions instead of proposition form.

4 1 {I => (J => K)} Λ (J => ⌐I) / {(I Λ J) => K} Λ (J => ⌐ I)}

2 {(I Λ J) => K} Λ (J => ⌐I) 1, Exp.

5 1 (R Λ S) => (⌐R v ⌐S) / (⌐R v ⌐S) => (R Λ S) .

2 (⌐R v ⌐S) => (R Λ S) 1,De.M.

6 1 (T v ⌐U) Λ {(W Λ ⌐V) => ⌐T}/ (T v ⌐U) Λ {(W=>(⌐V=>⌐T)}

2 (T v ⌐U) Λ {(W=>(⌐V=>⌐T)} 1, Exp.

7 1 (X v Y) Λ (⌐X v Z)/ (X v Y Λ ⌐X) v {(X v Y) Λ Z}

2 (X v Y Λ ⌐X) v {(X v Y) Λ Z} 1, Dist.

8 1 Z => (A => B) / Z => ⌐{ ⌐ (A => B)}

2 Z => ⌐{ ⌐ (A => B)} 1, D.N.

9 1 (⌐F v G) Λ (F => G) / F => G.)

2 (F => G) Λ (F => G) 1 Impl.

3 F=> G 2 Taut.

11.3 TESTING THE VALIDITY OF ARGUMENTS (THE RULES OF REPLACEMENT)

Now we shall consider different types of arguments, which may involve both kinds of rules. Although construction of formal proof is an interesting section in Symbolic Logic, certain tips as regards its procedure is in order. 1. Concentrate on the general form of the argument, and not to he confused by the complexity of the statements involved. See the following example:

(A v D) => [(C v D) => (C => D)]

⌐ [(C v D) => (C => D)]

∴⌐ (A v D)

Although it is a highly complex argument verbally and symbolically, closer observation will tell us that it is an instance of Modus Tollens. 2. Simplification will help us in dropping the statements; H.S. will drop the middle term and connect with a new consequent. M.P. liberates the consequent. 3. Distribution enables us to transform a conjunction into disjunction and vice versa. Double negation avoids the negation signs. 4. If conclusion to be followed is a disjunction, it can be derived in three ways, i.e., by applying C.D. or D.D.; deduce a statement and the apply Addition, and find out an implication, then turn to a Disjunction. 5. If the conclusion is a conditional statement, it can be found through H.S., or deduce a disjunction and then turn it by applying Material implication. Thought-out application of the rules and imagination is the best means of success in constructing formal proof.

10 1 (O => ⌐ P) Λ (P => Q)
2 Q => O
3 ⌐R => P / R
4 ⌐ Q v O 2, Impl.
5 O v ⌐ Q 4, Com.
6 (O => ⌐ P) Λ (⌐ Q => ⌐ P) 1, Trans,
7 ⌐P v ⌐P 6, 5, C.D.
8 ⌐P 7, Taut.
9 ⌐ ⌐ R 3, 8, M.T.
10 R 9, D.N.

11. 1 X => (Y => Z)
2 X => (A => B)
3 X Λ (Y v A)
4 ⌐Z / B
5 (X Λ Y) => Z 1, Exp.
6 (X Λ A) => B 2, Exp.
7 (X Λ Y) v (X Λ A) 3, Dist.
8 {(X Λ Y) => Z} Λ {(X Λ A) => B} 5,6, Conj.
9 Z v B 8, 7, C.D.
10 B 9, 4, D.S.

Notes

12. 1 C => (D => ⌐ C)

2 C ≡ D / ⌐C v ⌐D

3 C => (⌐ ⌐C => ⌐D) 1, Trans.

4 C => (C => ⌐D) 3, D.N.

5 (C Λ C) => ⌐D 4, Exp.

6 C => ⌐D 5, Taut.

7 ⌐C v ⌐D 6, Impl.

13. 1 E Λ (F v G)

2 (E Λ G) => ⌐ (H v I)

3 ⌐ (⌐H v ⌐I) => ⌐ (E Λ F) / H ≡ I

4 (E Λ G) => (⌐H Λ ⌐I) 2, De.M.

5 ⌐ (H Λ I) => ⌐ (E Λ F) 3, De.M.

6 (E Λ F) => (H Λ I) 5, Trans.

7 {(E Λ F) => (H Λ I)} Λ {(E Λ G) => (⌐H Λ ⌐I)} 6,4,Conj.

8 (E Λ F) v (E Λ G) 1, Dist.

9 (H Λ I) v (⌐H Λ ⌐I) 7,8, C.D.

10 H ≡ I 9, Equiv.

14. 1 J v (⌐K v J)

2 K v (⌐J v K) / J ≡ K

3 (⌐K v J) v J 1, Com.

4 ⌐k v (J v J) 3, Ass.

5 ⌐K v J 4, Taut.

6 K => J 5, Impl.

7 (⌐J v K) v K 2, Com.

8 ⌐J v (K v K) 7, Ass.

9 ⌐J v K 8, Taut.

10 J => K 9, Impl.

11 (J => K) Λ (K => J) 10, 6, Conj.

12 J ≡ K 11, Equi.

15. 1 (E Λ F) Λ G

2 (F ≡ G) => (H v I) / I v H

3 E Λ (F Λ G) 1, Ass.

4 (F Λ G) Λ E 3, Com.

5 (F Λ G) 4, Simp.

6 (F Λ G) v (¬ F Λ ¬ G) 5, Add.

7 F ≡ G 6, Equiv.

8 H v I 2, 7, M.P.

9 I v H 8, Com.

16. 1 (M => N) Λ (¬ O v P)

2 M v ¬ O / N v P

3 N v P 1, 2,C.D.

17. 1 (L v M) v (N Λ O)

2 (¬ L Λ O) Λ ¬ (¬ L Λ M) / ¬L Λ N

3 ¬ L Λ [O Λ ¬ (¬ L Λ M)] 2, Ass.

4 ¬ L 3, Simp.

5 L v {(M v (N Λ O)} 1, Ass.

6 M v (N Λ O) 5,4, D.S.

7 ¬ (¬ L Λ M) 2, Simpl.

8 Lv ¬ M 7, De. M.

9 ¬ M 8, 4, D.S.

10 N Λ O 6, 9, D.S.

11 N 10, Simpl.

12 ¬ L Λ N 4,11, Conj.

18. 1 E => (F => G) / F => (E => G)

2 (E Λ F) => G 1, Exp.

3 (F Λ E) => G 2, Com.

4 F => (E => G) 3, Exp.

19. 1 H => (I Λ J) / H=>I

2 ¬ H v (I Λ J) 1, Impl.

3 (¬ H v I) Λ (¬ H v J) 2, Dist.

4 ⌐ H v I 3, Simp.

5 H => I 4, Impl.

20. 1 N => O / (N Λ P) => O

2 ⌐ N v O 1, Impl.

3 ⌐ P v ⌐ N v O 2, Add.

4 ⌐ (P Λ N) v O 3, De.M.

5 (P Λ N) => O 4, Impl.

6 (N Λ P) => O 5, Com.

21. 1 (Q v R) => S / Q => S

2 ⌐ (Q v R) v S 1, Impl.

3 (⌐ Q Λ ⌐R) v S 2, De.M.

4 (⌐Q v S) Λ (⌐ R v S) 3, Dist.

5 ⌐Q v S 4, Simp.

6 Q => S 5, Impl.

22. 1 T => ⌐ (U => V) / T => U

2 T => ⌐ {⌐ (U Λ ⌐V)} 1, D.N.

3 ⌐T v (U Λ ⌐V) 2, Impl.

4 (⌐T v U) Λ (⌐T v ⌐V) 3, Dist.

5 ⌐T v U 4, Simp.

6 T => U 5, Impl.

23. 1 W => (X v ⌐Y) / W => (Y => X)

2 W => (⌐Y v X) 1, Com.

3 W => (Y => X) 2, Impl.

24. 1 H => (I v J)

2 ⌐I / H => J

3 ⌐H v (I v J) 1, Impl.

4 ⌐H v (J v I) 3, Com.

5 (⌐H v J) v I 4, Ass.

6 ⌐H v J 5, 2, D.S.

7 H=> J 6, Impl.

25. 1 (K v L) => ⌐ (M Λ N)

2 (⌐M v ⌐N) => (O ≡ P)

3 (O ≡ P) => (Q Λ R) / (L v K) => (R Λ Q)

4 (L v K) => ⌐ (M Λ N) 1, Com.

5 (L v K) => (⌐M v ⌐N) 4, De.M.

6 L v K) => (O ≡ P) 5, 2, H.S.

7 (L v K) => (Q Λ R) 6, 3, H.S.

8 (L v K) => (R Λ Q) 7, Com.

26. 1 (D Λ E) => F

2 (D => F) => G / E=>G

3 (E Λ D) => F 1, Com.

4 E => (D => F) 3, Exp.

5 E => G 4, 2, H.S.

27. 1 (H v I) => {J Λ (K Λ L)}

2 I / J Λ K

3 I v H 2, Add.

4 H v I 3, Com.

5 J Λ (K Λ L) 1, 4, M.P.

6 (J Λ K) Λ L 5, Ass.

7 J Λ K 6, Simp.

28. 1 (M v N) => (O Λ P)

2 ⌐O / ⌐M

3 ⌐O v ⌐P 2, Add.

4 ⌐ (O Λ P) 3, De.M.

5 ⌐ (M v N) 1, 4, M.T.

6 ⌐M Λ ⌐N 5, De.M.

7 ⌐M 6, Simp.

29. 1 T Λ (U v V)

2 T => {U => (W Λ X)}

3 (T Λ V) => ⌐ (W v X) / W ≡ X

4 (T Λ U) => (W Λ X) 2, Exp.

5 (T Λ V) => (⌐W Λ ⌐X) 3, De.M.

6 {(T Λ U) => (W Λ X)} Λ {(T ΛV) => (⌐W Λ ⌐X)} 4, 5, Conj.

7 (TΛU) v (TΛV) 1, Dist.

8 (W Λ X) v (⌐WΛ⌐X) 6,7, C.D.

9 W ≡ X 8, Taut.

30. 1 Y => Z

2 Z => {Y => (R Λ S)}

3 ⌐ (R Λ S) / ⌐Y

4 Y => {Y => (R Λ S)} 1,2, H.S.

5 (Y Λ Y) => (R Λ S) 4, Exp.

6 Y => (R Λ S) 5, Taut.

7 ⌐ Y 6, 3, M.T.

31. 1 A v B

2 C v D / {(A v B) Λ C} v {((A v B) Λ D)

3 (A v B) Λ (C v D) 1, 2,Conj.

4 {(A v B) Λ C} v {(A v B) Λ D} 3, Dist.

32. 1 (I v ⌐ ⌐J) Λ K

2 {⌐ L => ⌐ (K Λ J)} Λ {K => (I=> ⌐M)} / ⌐ (M Λ ⌐ L)

3 {(K Λ J) => L} Λ {K => (I => ⌐ M)} 2, Trans.

4 {(K Λ J) => L} Λ {(K Λ I) => ⌐ M} 3, Exp.

5 (I v J) Λ K 1, D.N.

6 K Λ (I v J) 5, Com.

7 (K Λ I) v (K Λ J) 6, Dist.

8 (K Λ J) v (K Λ I) 7, Com.

9 L v ⌐M 4, 8, C.D.

10 ⌐ M v L 9, Com.

11 ⌐ (M Λ ⌐ L) 10, De. M.

11.4 THE RULES OF INFERENCE AND REPLACEMENT

Inference rules for propositional logic

Some of the rules are known under multiple names. I have tried to list a few of the popular ones.

Modus Ponens (MP) / Implication Elimination

Form1. P→Q

2. P

⊢ 3. Q

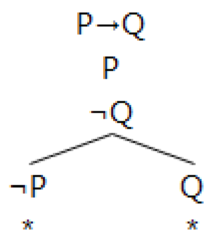
Example

1. If the earth exists, then a planet exists.

2. The earth exists.

Therefore, 3. A planet exists.

Proof tree



Modus Tollens (MT)

Form

1. P→Q

2. ¬Q

⊢ 3. ¬P

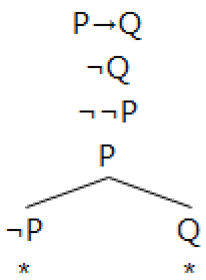
Example

1. If Socrates is a man, then Socrates is mortal.

2. Socrates is not mortal.

Therefore, 3. Socrates is not a man.

Proof tree



Hypothetical Syllogism (HS)

Form

1. P→Q

2. Q→R

⊢ 3. P→R

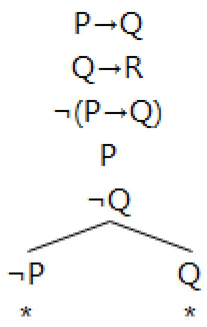
Example

1. If it rains, then the street is wet.

2. If the street is wet, then the street is slippery.

Therefore, 3. If it rains, then the street is slippery.

Proof tree



Disjunctive Syllogism (DS)

Form

1. P∨Q

2. ¬P

⊢ 3. Q

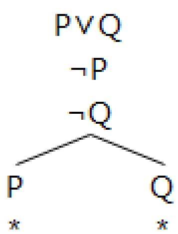
Example

1. Socrates is either male or female.

2. Socrates is not female.

Therefore, 3. Socrates is male.

Proof tree



Constructive Dilemma (CD)

Form

1. (P→Q)∧(R→S)

2. P∨R

⊢ 3. Q∨S

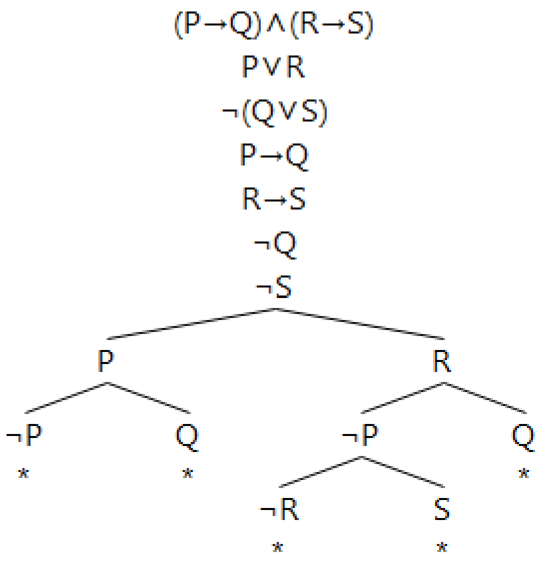
Example

1. If Socrates is a man, then Socrates is mortal, and is Plato is a man, then Plato is mortal.

2. Either Socrates or Plato is a man.

Therefore, 3. Either Socrates or Plato is mortal.

Proof tree



Destructive Dilemma (DD)

Form

1. (P→Q)∧(R→S)

2. ¬Q∧¬S

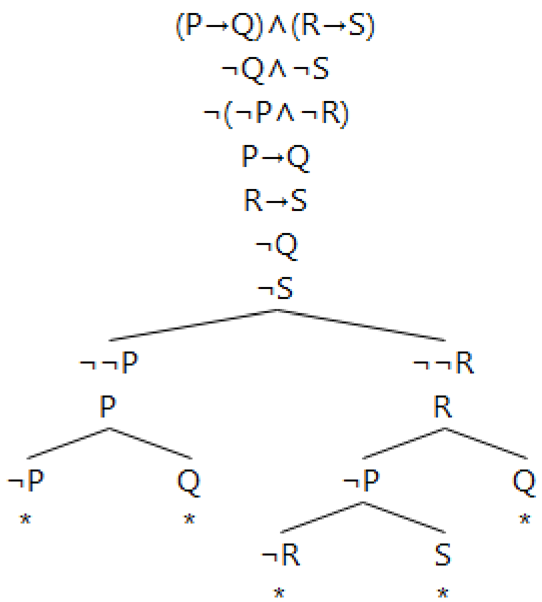
⊢ 3. ¬P∧¬R

1. If Socrates is a woman, then Socrates is dumb, and is Plato is a man, then Plato is dumb.

2. Socrates is not dumb and Plato is not dumb.

Therefore, 3. Socrates is not a woman and Plato is not a man.

Proof tree



Conjunction Introduction (Conj.) / Adjunction

Form

1. P

2. Q

⊢ 3. P∧Q

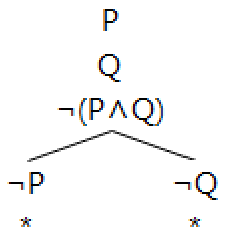
Example

1. It rains.

2. It is monday.

Therefore, 3. It rains and it is monday.

Proof tree



Simplification (Simp.) / Conjunction Elimination (CE)

Form

1. P∧Q

⊢ 2. P

Example

1. It is tuesday and the sun shines.

Therefore, 2. It is tuesday.

Proof tree

P∧Q
¬P
P
Q
*

Addition (Add.) / Disjunction Introduction (DI)

Form

1. P

⊢ 2. P∨Q

Example

1. I won the lottery.

Therefore, 2. I won the lottery or I am a woman.

Proof tree

P
¬(P∨Q)
¬P
¬Q
*

Replacement rules for propositional logic

I will ommit exmaples of these and just list the forms and proofs.

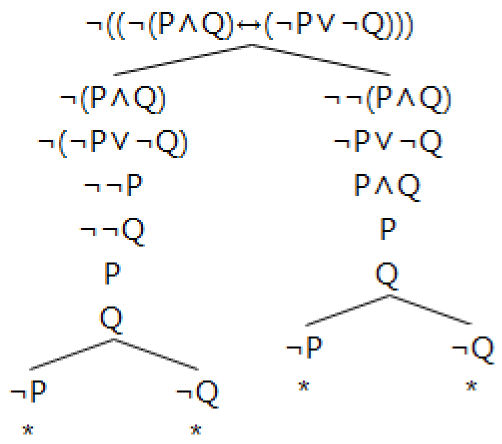
De Morgan's laws

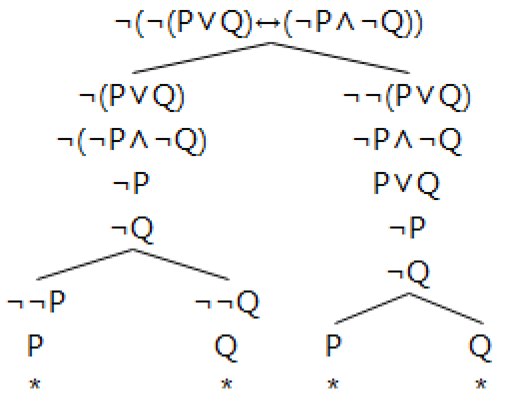
Forms

⊢ ¬(P∧Q)↔(¬P∨¬Q)

⊢ ¬(P∨Q)↔(¬P∧¬Q)

Proof trees



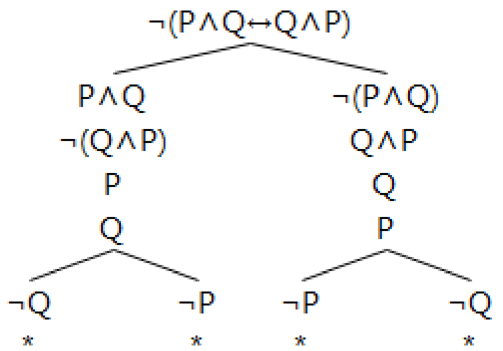


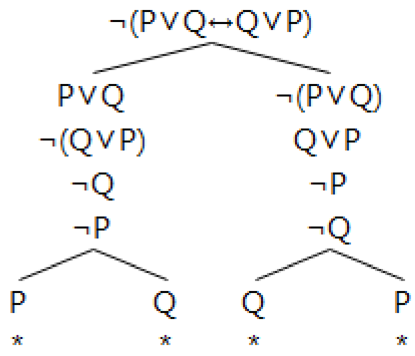
Commutation (Com.)

Forms⊢ P∧Q↔Q∧P

⊢ P∨Q↔Q∨P

Proof trees





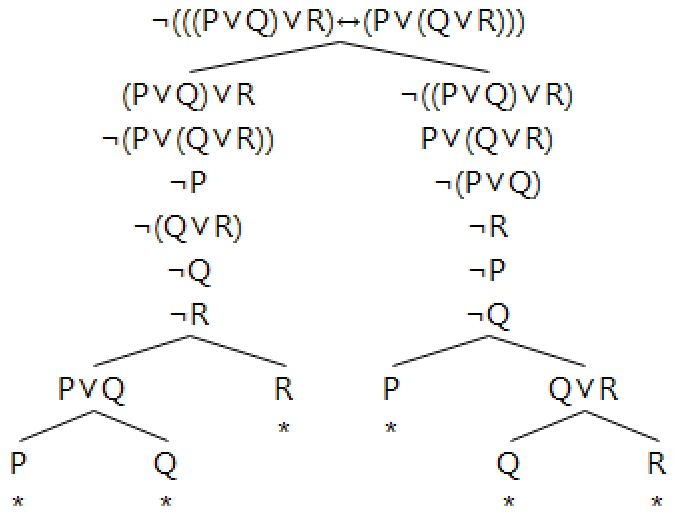
Association (Assoc.)

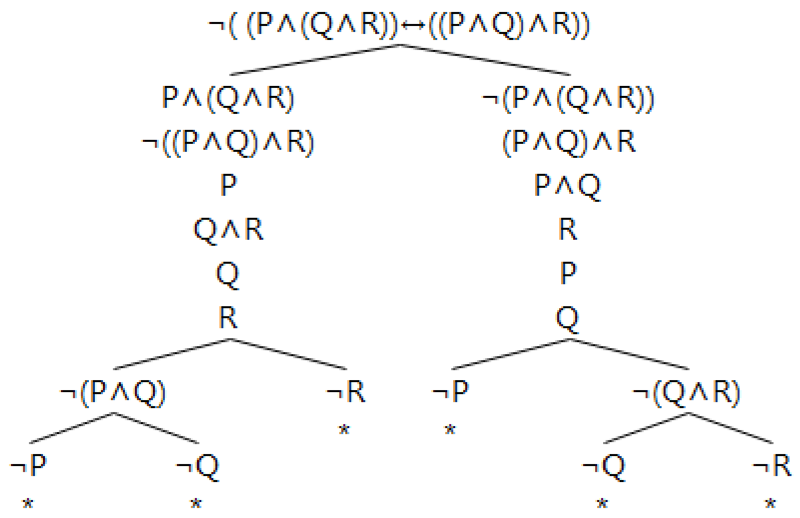
Forms

⊢ ((P∨Q)∨R)↔(P∨(Q∨R))

⊢ (P∧(Q∧R))↔((P∧Q)∧R)

Proof trees





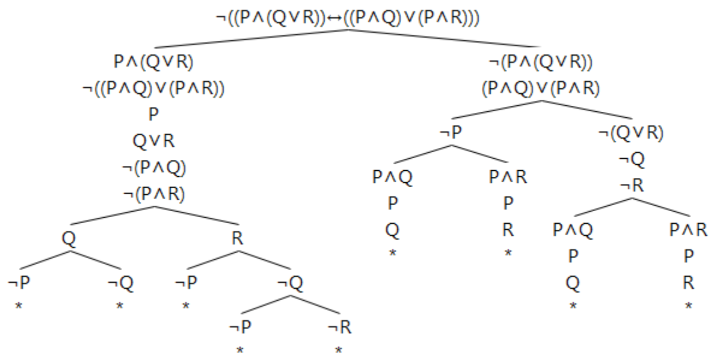
Distrubution (Dist.)

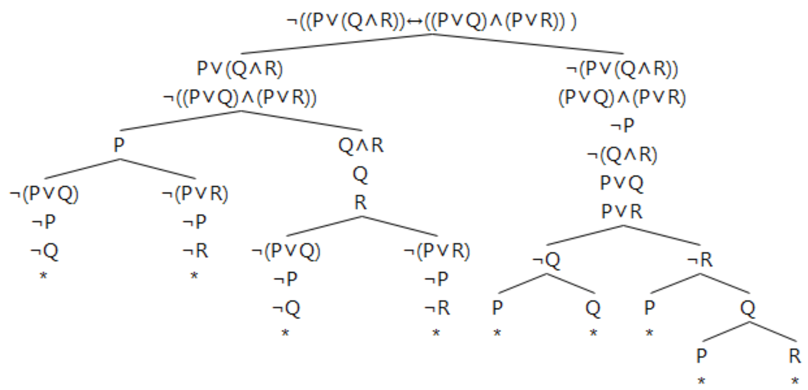
Forms

⊢ (P∧(Q∨R))↔((P∧Q)∨(P∧R))

⊢ (P∨(Q∧R))↔((P∨Q)∧(P∨R))

Proof trees



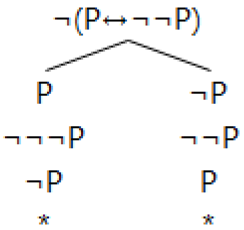


Double Negation (DN)

Form

⊢ $P \leftrightarrow \neg\neg P$

Proof tree



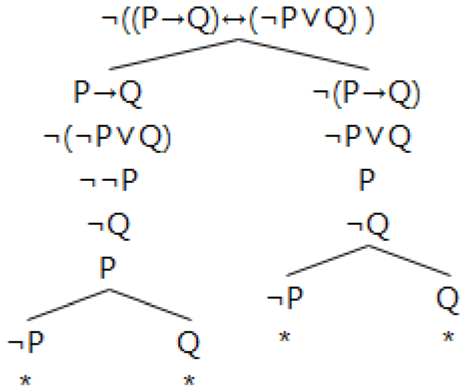
Material Implication (M. Imp.) / Material Conditional

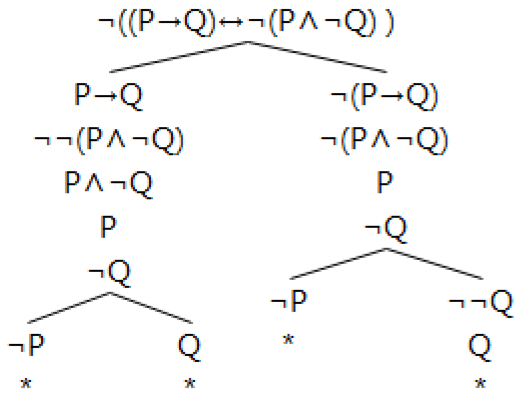
Forms

⊢ $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$

⊢ $(P \rightarrow Q) \leftrightarrow \neg(P \wedge \neg Q)$

Proof trees





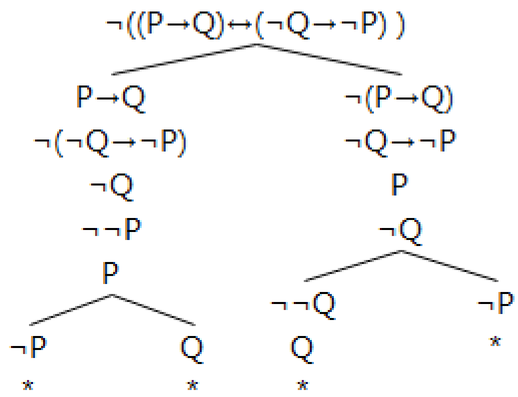
Transposition (Trans.) / Contraposition (CP.)

Notes

Form

⊢ (P→Q)↔(¬Q→¬P)

Proof tree



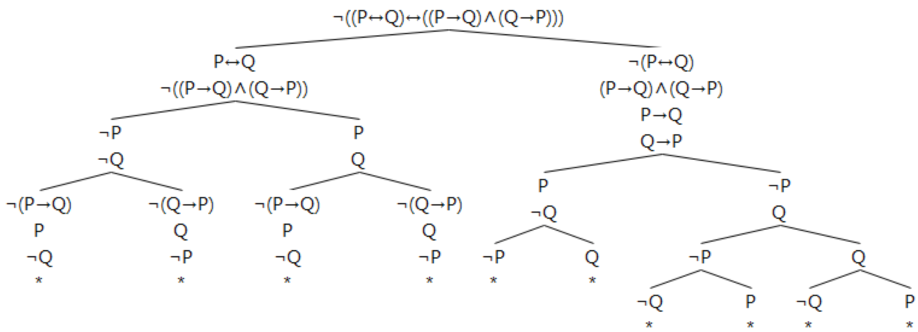
Material Bi-implication / Material Equivalence (M. Equiv.)

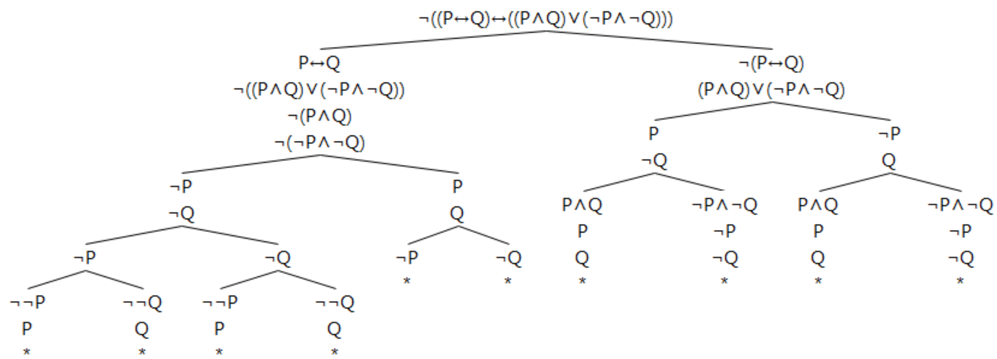
Forms

⊢ (P↔Q)↔((P→Q)∧(Q→P))

⊢ (P↔Q)↔((P∧Q)∨(¬P∧¬Q))

Proof trees

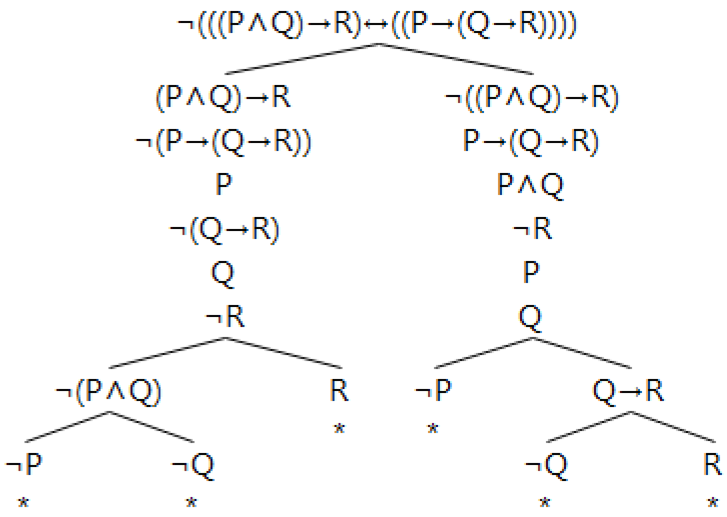




Exportation

Form⊢ ((P∧Q)→R)↔((P→(Q→R)))

Proof tree



Inference rules for predicate logic

Universal Instantiation (UI)

Form

1. ∀xFx

⊢ 2. Fa

Where "a" is some constant which ∀x quantifies over.

Example:

1. All humans have lungs.

Therefore 2. Socrates has lungs.

Proof tree

Notes

∀xFx

¬Fa

Fa

*

Existential Generalization (EG)

Form

1. Fa

⊢ 2. ∃xFx

Example:

1. Socrates has lungs.

⊢ 2. There is someone who has lungs.

Proof tree

Fa

¬∃xFx

∀x¬Fx

¬Fa

*

Replacement rules for predicate logic

Quantifier rules

⊢ ((∃x)(...))↔(¬(∀x)(¬(...)))

⊢ ((∀x)(...) ↔(¬(∃x)(¬(...)))

"..." can be replaced by whatever as long as it results in a wff. I used "Fx" in my proof.

Proof trees

$\neg(((\exists x)(Fx))\leftrightarrow\neg((\forall x)(\neg(Fx))))$

$(\exists x)Fx$	$\neg(\exists x)Fx$
$\neg\neg(\forall x)\neg Fx$	$\neg(\forall x)\neg Fx$
Fa	$\forall x\neg Fx$
$(\forall x)\neg Fx$	$\exists x\neg\neg Fx$
$\neg Fa$	$\neg\neg Fb$
*	Fb
	$\neg Fb$
	*

$\neg(((\forall x)(Fx))\leftrightarrow(\neg(\exists x)(\neg(Fx)))$

$(\forall x)Fx$	$\neg(\forall x)Fx$
$\neg\neg(\exists x)\neg Fx$	$\neg(\exists x)\neg Fx$
$(\exists x)\neg Fx$	$\exists x\neg Fx$
$\neg Fa$	$\forall x\neg\neg Fx$
Fa	$\neg Fb$
*	$\neg\neg Fb$
	Fb
	*

Let us start with verbal form of argument and symbolize the statement and logical constants before proceeding to test the validity of the arguments. (Problems are worked out at the end.)

1. Oxygen in the tube either combines with filament to form an oxide or else it vanishes completely. Oxygen in the tube could not have vanished completely. Therefore the oxygen in the tube combined with the filament to form an oxide.

2. If a political leader who sees her former opinions to be wrong does not alter her course, she is guilty of deceit; and if she does alter her course, she is open to a charge of inconsistency. She either alters her course or she does not. Therefore either she is guilty of deceit or else she is open to a charge of inconsistency.

3. It is not the case that she either forgot or wasn’t able to finish. She did not forget. Therefore she was able to finish.

4. She can have many friends only if she respects them as individuals. If she respects them as individuals, then she cannot expect them all to behave alike. She does have many friends. Therefore she does not expect them all to behave alike.

5. If the victim had money in his pockets, then robbery was not the motive for the crime. But robbery or vengeance was the motive for the crime. The victim had money in his pockets. Therefore vengeance must have been the motive for the crime.

6. Napoleon is to be condemned if he usurped power that was not rightfully his own. Either Napoleon was a legitimate monarch or else he usurped power that was not rightfully his own. Napoleon was not a legitimate monarch. So Napoleon is to be condemned.

7. If we extend further credit on the Wilkins account, they will have a moral obligation to accept our bid on their next project. We can figure a more generous margin of profit in preparing our estimates if they have a moral obligation to accept our bid on their next project. Figuring a more generous margin of profit in preparing our estimates will cause our general financial condition to improve considerably. Hence a considerable improvement in our general financial condition will follow from our extension of further credit on the Wilkins account.

8. Had Roman citizenship guaranteed civil liberties, then Roman citizens would have enjoyed religious freedom. Had Roman citizens enjoyed religious freedom, there would have been no persecution of the early Christians. But the early Christians were persecuted. Hence Roman citizenship would not have guaranteed civil liberties.

9. Jalaja will come if she gets the message provided that she is still interested. Although she did not come she is still interested. Therefore she did not get the message.

10. If the teller or the cashier had pushed the alarm button, the vault would have locked automatically and the police would have arrived within three minutes. Had the police arrived within three minutes, the robber's car would have been overtaken. But the robber's car was not overtaken. Therefore the teller did not push alarm button.

11. If people are always guided by their sense of duty, they forget the enjoyment of many pleasures; and if they are always guided by their desire for pleasure, they must often neglect their duty. People are either always guided by their sense of duty or always guided by their desire for pleasure. If people are always guided by their sense of duty, they do not often neglect their duty; and if they are always guided by their desire for pleasure, they do not forget for enjoyment of many pleasures. Therefore people must forget the enjoyment of many pleasures if and only if they do not often neglect their duty.

12. Although world population is increasing agricultural production is declining and manufacturing output remains constant. If agricultural production declines and world population increases, then either new food sources will become available or else there will be a radical redistribution of food resources in the world unless human nutritional requirements diminish. No new food sources will become available, yet neither will family planning be encouraged nor will human nutritional requirements diminish. Therefore there will be a radical redistribution of food resources in the world.

Answers: The components are symbolized in this way:

1. 1 Combined with filament: C

2 Else it vanished: V

3 Could not have vanished: ¬V

Statements / Arguments:

1 C v V

2 ¬V / C 1, D. S.

2. 1 A political leader…does not alter her course: ¬C

2 She is guilty of deceit: D

3 She alters her course: C

4 She is open to a charge of inconsistency I

Statements:

1 (⌐ C => D) Λ (C => I)

2 C v ⌐ C / D v I

3 ⌐ C v C 2, Com.

4 D v I 1,3, C.D.

3. 1 It is not the case that: ⌐

2 She either forgets: F

3 She did not forget. ⌐ F

3 Not able to furnish: ⌐A

Statements / Arguments

1 ⌐ (F v ⌐A)

2 ⌐ F / A

3 ⌐ F Λ A 1,De.M.

4 A 3,Simpl.

4. She does not respect them as individuals: ⌐ R

She can have many friends: F

She cannot expect… ⌐ E

Statement / Argument

1 ⌐R => ⌐ F

2 R => ⌐ E

3 F / ⌐ E

4 R 1, 3, M.T.

5 ⌐ E 2, 4, M.P.

5. 1 The victim had money… M

2 Robbery was not the motive ⌐R

3 Robbery or Vengence…. R v V

Statements / Argument

1 M => ⌐R

2 R v V

3 M / V

4 ⌐ R 1, 3, M.P.

V 2, 4, D.S.

6. 1 He usurped power that was not…. U

2 Napoleon is to be condemned C

3 Napoleon was a legitimate… L

Statements / Argument

1 U => C

2 L v U

3 ⌐ L / C

4 U 2,3, D.S.

C 1, 4, M.P.

7. 1 We extend further credit… C

2 They will have a moral obligation…. M

3 We can figure…. F

4 Considerable improvement… I

Statement / Argument

1 C => M

2 M => F

3 F => I / C => I

4 C => F 1,2, H.S.

C => I 4,3, H.S.

Check Your Progress

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

Out of several problems, we have worked out seven problems. The student is advised to solve the rest, which is a very good method of learning to test the arguments of complicated structure.

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

……………………

11.5 TEST OF ARGUMENTS IN VERBAL FORM

A strong argument is that which will touch the practical as well as the real aspect of the situation as mentioned in the statements. While in a weak argument, the statement drawn will be simple, ambiguous, and a superfluous one. The following solved examples will help you understand better, the logic and types of questions that are generally asked in the competitive exams. Also, there are practice questions at the end of the examples.

Before the examples, there are points that you need to keep in mind and remember while solving and practicing these questions.

1. While you make a decision about the important questions, it is desirable to be able to differentiate between 'strong' and 'weak' arguments so far as they relate to the questions.
2. For 'weak' arguments, the important thing to note is that they may or may not be directly related to the question and may be of minor importance.
3. Also, they may be related to trivial part of the question. The questions given in this topic will be followed by two statements I and II.
4. Based on the question, you have to determine which argument is strong and which one is weak.

Solved Examples

For all the questions below, you have to answer based on these sentences.

a) the only argument I am strong

b) if only argument II is strong

c) neither I nor II is strong

d) if both I and II are strong.

Statement: Should the schooling education be made free in India?

Arguments

I: Yes, this is the only way through which we can improve the level of literacy.

II: No, this will add to the already heavy burden economy of India.

In this questions, to make the argument weak or strong you need to look for small clues and ideas. For example, in the statement I the use of word 'only' is not strong enough and thus makes the argument weak. If you can see, it is not the only real and practical solution to improve the level of literacy.

Meanwhile, in comparison to the argument I, argument II is strong enough because it describes the practical problem which may happen due to the decision was taken for making the schooling education free. Thus argument II is the strong argument. So, the correct answer is B.

In statements and arguments, you need to use your logic. Instead, work only with the information provided in the statement. Do not try and go for a practical approach.

Statement: Should the crackers be completely banned in India?

Arguments.

I: Yes, the use of child labours in the manufacturing of firecrackers is very high.

II: No, the jobs of thousand workers will be hindered.

In this question, there is no morally correct or incorrect approach. Both the statements refer to the practical consequences of the action being taken for the statement given in the question. Thus both the arguments are given in the question is correct. So, the correct answer is D.

11.6 LET US SUM UP

Just as the laws of traditional logic are inadequate to test the validity, rules of inference also are inadequate. So the stock of rules is further augmented with the help of the rules of replacement. Rule of inference applies to the whole line. However, the rule of replacement may apply to the whole line or any part of the line. Various types of arguments can be tested with the help of these rules.

11.7 KEY WORDS

Usurp: Usurp is to seize power from another, usually by illegitimate means.

Technology: Technology is a broad concept that deals with human's usage and knowledge of tools and crafts, and how it affects human's ability to control and adapt to environment.

Technology is a term with origins in the Greek "technologia," "techne" ("craft") and "logia" ("saying"). However, a strict definition is elusive; "technology" can refer to material objects of use to humanity, such as machines, hardware or utensils, but can also encompass broader themes, including systems, methods of organization, and techniques.

11.8 QUESTIONS FOR REVIEW

1. Discuss the Rules of Replacement in Symbolic Logic: Formal Proof of Validity.
2. Out of several problems, we have worked out seven problems. The student is advised to solve the rest, which is a very good method of learning to test the arguments of complicated structure.

11.9 SUGGESTED READINGS AND REFERENCES

Copi, I.M. Symbolic Logic. 4th Ed. New Delhi: Collier Macmillan International, 1973.

Copi, I.M. Introduction to Logic. 9th Ed. New Delhi: Prentice Hall of India, 1995.

Joseph, H.W.B. An Introduction to Logic. Oxford: 1906.

Lewis, C.I. & Longford, C.H. Symbolic Logic. New York: Dover Pub. Inc.,1959.

11.10 ANSWERS TO CHECK YOUR PROGRESS

It may be noted that every symbol is the first letter first or second term in the respective component.

8 1) R => F
2) F => ⌐ C
3) C / ⌐ R
4) ⌐ F 2, 3, M.T.
5) ⌐ R 1, 4, M.T.

9 This particular argument is in need of some restructuring for the sake of convenience without
changing meaning. It is done in the following manner.

If Jalaja will come and she is interested then she would have got the message. She did not come
and she is interested. Therefore she did not get the message.
Now it is easy to symbolize.

1) (J Λ I) => S
2) ⌐ S Λ I / ⌐ J
3) J => (I =>S) 1,Exp.
4) J => (⌐ I v S) 3,Impl.
5) J => ⌐ (I Λ ⌐ S) 4,De.M.
6) I Λ ⌐S 2,Com.
7) ⌐J 5,6,M.T.

10

Notes

1) (T v C) => (V Λ P)

2) P => R

3) ⌐ R / ⌐ T

4) ⌐ P 2, 3, M.T.

5 ⌐ P v ⌐V 4, Add.

6 ⌐V v ⌐ P 5,Com.

7 ⌐ (T v C) 1,6. M.T

8 ⌐ T Λ ⌐ C 7, De.M.

9 ⌐T 8, Simp.

11

1 (D => F) Λ (P => N)

2 D v P)

3 (D => ⌐ N) Λ (P => ⌐ F) / (F => ⌐ N) Λ (⌐ N => F)

4 F v N 1, 2, C.D.

5 ⌐ N v ⌐ F 3, 2, C. D.

6 ⌐ F => N 4, Impl.

7 N => ⌐ F 5,Impl.

8 (⌐ F => N) Λ (N => ⌐ F) 6, 7, Conj.

9 (⌐ N => F) Λ (F => ⌐N) 8, Trans.

10 (F => ⌐ N) Λ (⌐ N => F) 9, Com.

12 This argument also stands in need of restructuring of some sentences. It runs as follows. World population is increasing and agricultural production is declining and manufacturing output remains constant. When symbolized it becomes W and A and M Next the phrase ‘unless human nutritional requirements diminish’ becomes human nutritional requirements do not diminish’. And then the statement ‘neither will family planning be encouraged nor will human nutritional requirements diminish’ means the same as world population is increasing and human nutritional requirements do not diminish. The next stage is now obvious. The whole argument can be symbolized.

1 W Λ (A Λ M)

2 (A Λ W) => (N v R) Λ H

3 ⌐ N

4 W

5 ⌐ H / R

6 (W Λ A) Λ M 1, Ass.

7 W Λ A 6, Simp.

8 (A Λ W) 7, Com.

9 (N v R) Λ H 2, 8, M.P.

10 N v R 9, Simp.

11 R 10, 3, D.S.

Note that line 5 is redundant though it is the part of the argument. Therefore it can be ignored.

UNIT 12: THE LOGIC OF RELATION

STRUCTURE

12.0 OBJECTIVES

In this unit 12 propose to introduce a new list of techniques of testing the validity of arguments. There are as many kinds of techniques as there are arguments. The main purpose of this unit is to make you understand that there is not a single technique which helps you to solve all kinds of problems. It is not sufficient if you know the art of testing validity only. Therefore one of the aims is to introduce you to the art of testing invalidity also. To have a satisfactory knowledge of good argument you should also know what makes an argument bad. Therefore this unit introduces you to this aspect of the study of logic.

12.1 INTRODUCTION

The method of Conditional Proof (C.P.) is different in kind from the rules of inference or replacements. There are a certain types of arguments, which cannot be tested with any of the rules discussed in the previous chapters without further support. The rules discussed earlier are restricted only to those arguments, which have unconditional conclusions. So an argument, which has conditional conclusion, falls out of their purview. The most familiar example for conditional proposition is implicative proposition. Since implicative propositions have equivalent disjunctive and negation forms, they are also to be regarded as conditional propositions. Again, C.P is not a system of proof, which does away with the nineteen rules. Only, the number increases to twenty. Among them one rule is compulsorily used to test the validity when the conclusion is conditional. This rule is characteristic of C.P in the sense that nowhere else it is used. Hence this rule can be designated as the rule of C.P.

12.2 LOGIC OF RELATION

We have now examined the philosophical framework surrounding De Morgan’s views on relations, and we have also seen how these views show the need for a logic of relations. In this unit, we will discuss De Morgan’s central contribution to the logic of relations, which he published in 1860 under the title, “On the Syllogism: IV and on the Logic of Relations.” In this classic memoir, De Morgan moves beyond his relational analysis of the syllogism and the bicopular syllogism to something that may justifiably be called a logic of relations: that is, the specification and systematization of previously unrecognized valid forms of relational inference.

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12.3 CONDITIONAL PROOF

Any deductive argument, whether valid or invalid, can be expressed in the form of a conditional proposition. What is more important to know is that the original argument is valid only when the corresponding conditional statement fulfills a condition known as ‘tautology’. Otherwise the argument is invalid. Consider this example: 1). All A are B All B are C / ∴ All A are C Its corresponding conditional form is as follows: “If all A are B and all A are C, then all A are C”. (1). Let the first premise be symbolized as P1 and second as P2. Conclusion is symbolized as C. Now (1) becomes: (P1Λ P2) => C (2) (2) is said to be tautologous because its corresponding proposition form is tautologous. A proposition form is said to be tautologous when it has only true substitution. No matter how many substitutions we make for proposition form, all of them must be true. In other words, if there are ‘n’ number of instances in which substitution is made to the proposition form, then in all these ‘n’ instances the proposition form must be true. There are two conditions to be satisfied if C. P. should show that the argument is valid.

1) Conclusion must be a conditional proposition.

2) It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments which satisfy the relevant rules of inference.

That is, all premises in C.P. should be supported by these rules. The additional premise, which is a characteristic mark of C.P., is always the antecedent of the conclusion and the construction of proof always begins with antecedent of the conclusion as the premise. This premise itself is called C.P. An example of argument, which requires C.P., is given below. (3) P => (A=> B) When P stands for the conjunction of premises, one of the rules of replacement, i.e., exportation rule permits us to rewrite (3) as: (4) (PΛ A) => B It is obvious that the conclusion of (4) is the consequent of the conclusion of (3). Since we start with an assumed premise, the proof is known as C.P. Here is the difference. All other premises are taken as true. The assumption should not really matter. Even if the assumed premise is false, it is possible to deduce valid conclusion. If B can be validly drawn from P and A then not only (A) is valid its corresponding original argument (3) also must be valid because (3) and (4) are logically equivalent argument of this form.

1. 1. (A v B)=> (CΛ D)
2). (D v E) => F / ∴ A=>F
We should start from assuming A.
3). A /∴ F C. P.

In C. P. always the first line must have this structure. Slash against line 3 in, ∴
and (C.P) indicate that the method of conditional proof is being used.

4). A v B 3, Add.
5). C Λ D 1, 4, M.P.
6). D 5, Simp.
7). D v E 6, Add.

8). ∴ F 2, 7, M.P.

If there is only one condition in the conclusion, then C.P is used once. If there are two conditions in the conclusion, then C.P. is used twice. In such cases the procedure to be followed is as follows.

2. 1). A=> (B=> C)
2). B=> (C=> D) /∴ A => (B =>D)
3). A /∴ B => D (C.P.)
4). B /∴ D (C.P.)
5). B => C 1, 4, M.P.
6). C 5, 4, M.P.
7). C => D 2, 4, M.P.
8). ∴D 7, 6, M.P.

12.4 INDIRECT PROOF

This method is also known as reductio ad absurdum, a method very common in the construction of proof of geometrical theorems. This method is characterized by a special feature. In order to prove a certain statement, its contradiction is assumed to be true from which the conclusion, which contradicts our assumption, is logically deduced. If A contradicts ⌐ B, then either A must be false or ⌐ B must be false. A cannot be false because it is logically deduced from what is purported to be true. Therefore ⌐B must be false, which means that B must be true. This is how a theorem in geometry or an argument in logic is, sometimes, proved. This method has a distinct advantage. Sometimes the length of proof is too long. In logic it is important that we use the least number of steps. Second requirement is clarity. Combination of these two is what is most desired. In such circumstances, this method is most useful. The use of this method consists in beginning with the contradiction of what is to be proved. A point to be noted here is that, the contradiction of what has to be proved is marked by writing I.P. on the right hand side just adjacent to the assumption. In C.P. also we begin with assumption. The difference is that in the latter what is assumed is a part of the argument whereas in the case of former it is not. Consider this argument.

1. A => (BΛC)
2. (B v D) => E
3. D v A / ∴ E
4. ⌐ E I.P.
5. ⌐B Λ ⌐ D 2, 4, M.T.
6. ⌐D 5, Simp.
7. A 3, 6, D.S.
8. BΛC 1, 7, M.P.
9. B 8, Simp.
10. B v D 9, Add.
11. E 2, 10, M.P.
12. E Λ ⌐ E 11, 4, Conj.

Tenth Step can also be written and consequent step in this manner

13. ⌐ B 5, Simp.
14. B Λ ⌐ B 9, 13, Conj.

Whether we get E Λ ⌐ E or B Λ ⌐ B, the result remains the same. In both the cases there are steps in the argument whose conjunction leads to contradiction. Wherever there is contradiction, one conjunct must be false so that the other one has to be true.

12.5 THE STRENGTHENED RULE OF CONDITIONAL PROOF

In Conditional Proof method, the conclusion depends upon the antecedent of the conclusion. There is another method, which is called the strengthened rule of conditional proof. In this method, the construction of proof does not necessarily assume the antecedent of the conclusion. The structure of this method needs some elaboration. An assumption is made initially. There is no need to know the truth-status of the assumption because an assumption may be false, but the conclusion can still be true. Further, the assumption can be any component of any premise or conclusion. This method is called the strengthened rule because we enjoy more freedom in making assumption or assumptions, which means that plurality of assumptions is allowed. It strengthens our repertoire of testing equipments. In this sense, this method is called the strengthened rule of C.P. Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more than two assumptions in an argument, then there will be a distinct last step with respect to each assumption. This last step can be regarded as the conclusion relative to that particular assumption. It shows that the last step is deduced with the help of assumption in conjunction with the previous steps in such a way that the rules of inference permit such conjunction. Before the conclusion is reached the function of assumption also ceases. Then it will have no role to play. Then, automatically, the assumption is said to have been discharged. When the strengthened rule of C. P. is used adjacent to the line of assumption, the word assumption is not mentioned unlike in the case of C.P. here the head of the bent arrow points to 'assumption'. In case of the strengthened rule of C.P., the conclusion is always a conditional statement which consists of statements from the sequence itself. Thus the range of the application of condition is defined. In order to easily identify the range of its application, a slightly different method is used. An arrow is used to indicate what is assumed and with the help of the same arrow its range also is defined. The application of C.P. in restricted to the space covered by the arrows. All steps, which are outside this arrow, are also independent of the condition. While the head of the arrow marks the assumption, its terminus separates the lines, which depend upon the condition from the line, which does not depend on the condition. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first premise only. In this sense, it is a strengthened condition. In this case there is no reason to mention C.P. because the arrow helps us to identify the assumption. Consider this example:

1.	(A v B) => {(C v D) => E} /∴ A=> [(C Λ D) => E]		
2.	A		
3.	A v B	2,	Add.
4.	(C v D) => E	1, 3,	M.P.
5.	(CΛD)		
6.	C	5,	Simp.
7.	C v D	6,	Add.
8.	E	4, 7,	M.P.
9.	(C v D) => E	5, 8,	C.P.
10.	A => [CΛD) => E]	2, 9,	C.P.

Rules mentioned on the RHS make it clear that all lines from 3 to 9 depend on A either directly or through bent arrows. In lines 9 and 10 implication makes them C.P. One advantage of C.P. in its strengthened form is that it has an extended application. It can be used in all those cases where conclusions are conditional, but do not appear to be so.

12.6 PROVING INVALIDITY

Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation of a rule or rules makes arguments invalid. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. Therefore in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line as it happens in the case of truth-table. Let us consider some examples.

1. E=> (F v G)
2. G => (H Λ I)
3. ⌐ H /∴E => I

1	1	0	0	0	1
E	F	G	H	I	⌐H

While following this method '0' should be assigned to the conclusion making the premises true. If this combination cannot be achieved, then the argument is valid, i.e., even after making the conclusion 0 if the premises cannot take the value 1, then the argument is valid. The components of conclusion and premises should be paired properly to carry out the test.

1. J => (K => L)
2. K=> (¬ L=> M)
3. (L v M) => N /∴ J => N

1	0	1	0	0	0
J	K	L	¬L	M	N

Here the conclusion is '0' whereas the combination of premises is 1. Hence the argument is invalid.

12.7 SYMBOLIZING RELATIONS

1. Symbolizing Relations Propositions with two or more proper names (of individuals) Example:

• Lincoln and Grant were presidents. Lincoln was a president and Grant was a president.

• Lincoln and Grant were acquainted.

2. Propositions that express relations between two individuals are called "binary or dyadic relations" Examples: John loves Mary. Plato was a student of Socrates. Isaac was a son of Abraham. New York is east of Chicago. Chicago is smaller than New York.

3. Propositions that express relations between three individuals are called "ternary or triadic relations" Examples: Detroit is between New York and Chicago. Helen introduced John to Mary. America won the Philippines from Spain.

4. Propositions that express relations between four individuals are called "quaternary or tetra decorrelations" Examples: America bought Alaska from Russia for seven million dollars. Jack traded his cow to the peddler for a handful of beans. Al, Bill, Charlie and Doug played bridge together.

5. Examples: Aristotle is human. Plato is human. Socrates is human. Al is older than Bill. Bill is older than Charlie. Therefore, Al is older than Charlie. Helen likes David. Whoever likes David likes Tom. Helen likes only good-looking men. Therefore, Tom is a good-looking man.

6. The active voice is the "normal" voice. This is the voice that we use most of the time. You are probably already familiar with the active voice. In the active voice, the object receives the action of the verb: active subject verb object > Cats eat fish.

7. The passive voice is less usual. In the passive voice,the subject receives the action of the verb: passive subject verb object < Fish are eaten by cats.

8. Examples: A attracts everything. Everything is attracted by A.A attracts something. Something is attracted by A. Everything attracts a. A is attracted by everything. Something attracts A.A is attracted by something.

9. 1. Everything attracts everything.2. Everything is attracted by everything.3. Something attracts something.4. Something is attracted by something.5. Nothing attracts anything.6. Nothing is attracted by anything.7. Everything attracts something.8. Something is attracted by everything.

10. Relational Proposition• They were simple-predicate assertions. ex: „a was struck‟ that can interpreted as „something struck a‟. (эx)(эy)(x struck a) or (эx)Sxb• They were also marked by the passive voice of a transitive verb.

11. Symbolizing of Proposition Purpose:-is to put then into a form convenient for testing their validity. Goal:-not to provide a theoretically complete analysis but to provide one complete enough for the purpose at hand- the testing of validity.

12. • Example: Whoever visited the building was observed. Anyone who had observed Andrews would have remembered him. Nobody remembered Andrews. Therefore, Andrews didn’t visit the building.

13. Unlimited Generality• Asserted that everything stood in such a relation or something did or nothing did. Ex: Everything is attracted by all magnets Mx= “x is a magnet” Axy= “x attracts y” (x)(y)(My»Axy)

14. Translating Relational Propositions into Logical Symbolism (limited generality)• Example: Any good amateur can beat some professional Sol: (x){(x is a good amateur)»(x can beat some professional)} The consequent of the conditional between the braces x can beat some professional Is symbolized as a quantified expressions (эy)[(y is a professional)•(x can beat y) Answer: (x){Gx»(эy)(Py•Bxy)]

12.8 ARGUMENTS INVOLVING RELATIONS

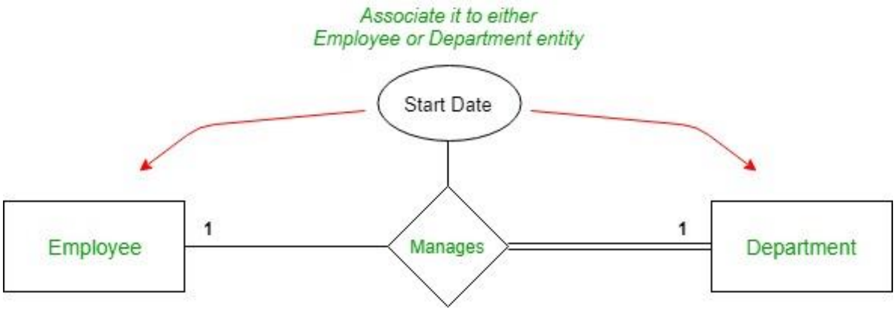
In Arguments as Relations, John Bowers proposes a radically new approach to argument structure that has the potential to unify data from a wide range of different language types in terms of a simple and universal syntactic structure. In many ways, Bowers's theory is the natural extension of three leading ideas in the literature: the minimalist approach to Case theory (particularly Chomsky's idea that Case is assigned under the Agree function relation); the idea of introducing arguments in specifiers of functional categories rather than in projections of lexical categories; and the neo-Davidsonian approach to argument structure represented in the work of Parsons and others. Bowers pulls together these strands in the literature and shapes them into a unified theory.

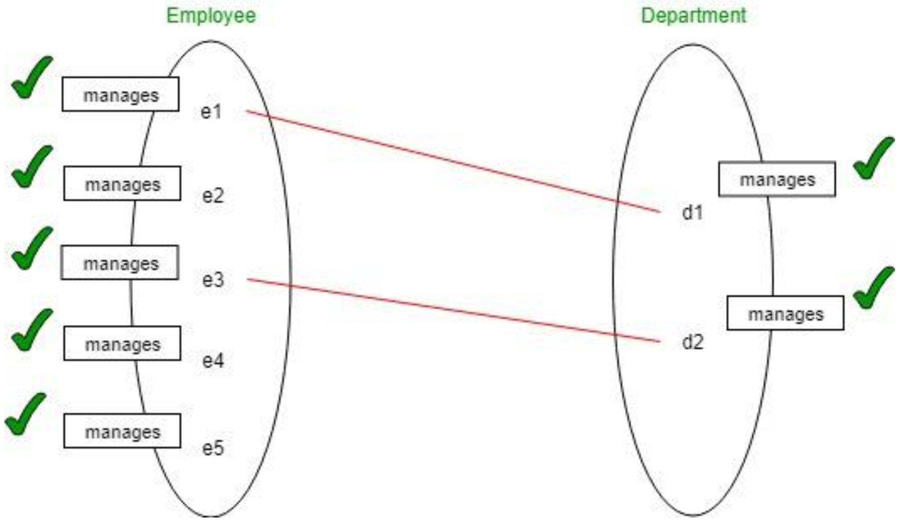
These ideas, together with certain basic assumptions—notably the idea that the initial order of merge of the three basic argument categories of Agent, Theme, and Affectee is just the opposite of what has been almost universally assumed in the literature—lead Bowers to a fundamental rethinking of argument structure. He proposes that every argument is merged as the specifier of a particular type of light verb category and that these functional argument categories merge in bottom-to-top fashion in accordance with a fixed Universal Order of Merge (UOM). In the hierarchical structures that result from these operations, Affectee arguments will be highest, Theme arguments next highest, and Agent arguments lowest—exactly the opposite of the usual assumption.

12.9 ATTRIBUTES OF RELATIONS

Notes

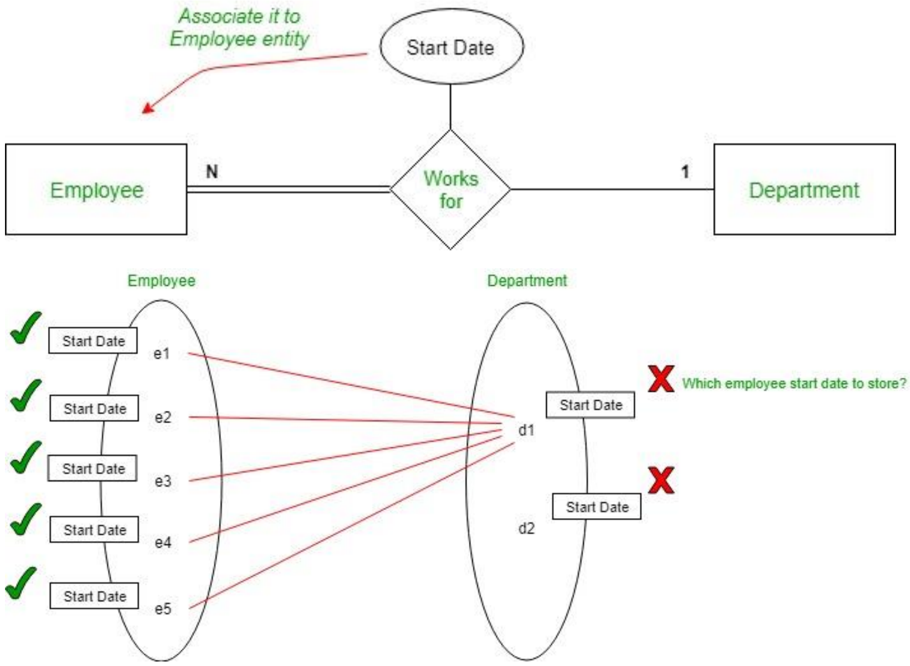
In an organisation an employee manages a department and each department is managed by some employee. So, there is a total participation of em>Department entity and there is *one to one* relationship between the given entities. Now, if we want to store the *Start_Date* from which the employee started managing the department then we may think that we can give the *Start_Date* attribute to the relationship *manages*. But, in this case we may avoid it by associating the *Start_Date* attribute to
either *Employee* or *Department* entity.





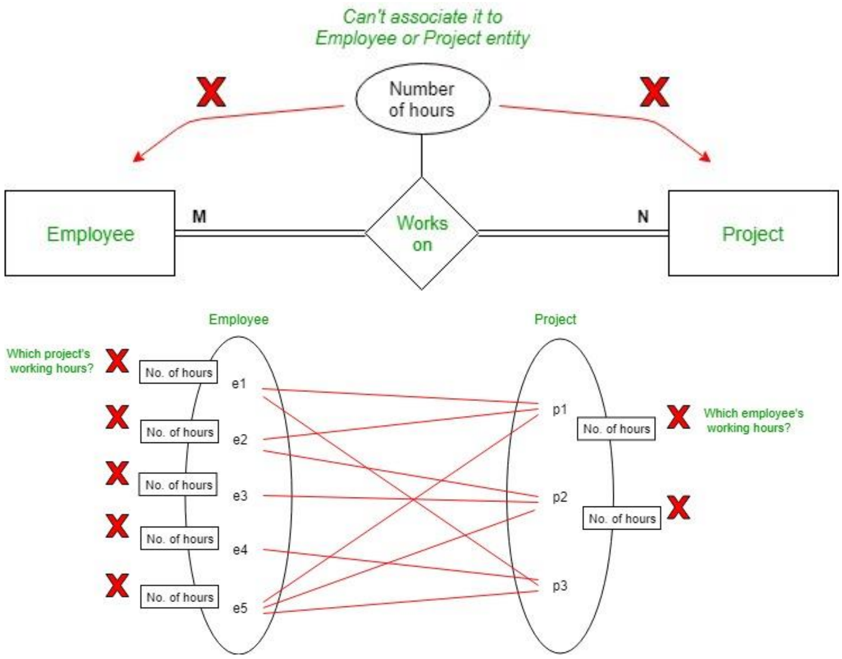
2. One to many relationship:

In an organisation many employees can work for a department but each employee can work for only a single department. So, there is a *one to many* relationship between the entities. Now if we want to store the *Start_Date* when employee started working for the department, then instead of assigning it to the relationship we should assign it to the *Employee* entity. Assigning it to the *employee* entity makes sense as each employee can work for only single department but on the other hand one department can have many employees working under it and hence, it wouldn't make sense if we assign Start_Date attribute to Department.



3. Many to many relationship:

In an organisation an employee can work on many projects simultaneously and each project can have many employees working on it. Hence, it's a *many to many* relationship. So here assigning the *Number_of_Working_hours* to the employee will not work as the question will be that it will store which project's working hours because a single employee can work on multiple projects. Similar the case with the *project* entity. Hence, we are forced to assign the *Number_of_Working_hours* attribute to the relationship.



12.10 EXERCISES

I Here some arguments are given which are tested using the method of C. P.

1

1. P Λ (Q => R) /∴ (P => Q) => (P => R)

2. P => Q /∴ P => R C.P.

3. P /∴ R C.P.

4. (P=> Q) => R 1, Exp.

5. ∴R 4, 2, M.P.

2

1. P => (Q => R) /∴ Q => (P => R)

2. Q /∴ P => R C.P.

3. P /∴ R C.P.

4. Q => R 1, 3, M.P.

5. ∴ R 4, 2, M.P.

,

3 1 A=> B / ∴ (B => C) => (A => C)

2 B => C /∴ A => C C.P.

3 A /∴ C C.P.

4 B 1, 3, M.P.

!!!6!∴ C 2, 4, M.P.

5

1. (A => B) Λ (A => C) /≵ A => (B v C)

2. A /≵ B v C C.P.

3. A=> B 1, Simp.

4. B 3, 2, M.P.

5. A=> C 1, Simp

6. C 5, 2, M.P.

7. ≵ B v C 4, Add.

6

1. (A=> B) Λ (A=> C) /A=> (BΛC)

2. A /≵ B Λ C C.P.

3. A => B 1, Simp.

4. B 3, 2, M.P.

5. A => C 1, Simp.

6. C 5, 2, M.P.

7. ≵ B Λ C 4, 6, Conj.

7

1. (A => B) /≵ (AΛC) => (BΛC)

2. A Λ C /≵ B Λ C C.P.

3. A 2, Simp.

4. B 1, 3, M.P.

5. C 2, Simp.

6. ≵ B Λ C 4, 5, Conj.

8

1. B => C /≵ (A v B) => (C v A)

2. A v B /≵ C v A C.P.

3. ⌐ A=> B 2, Impl.

4. ⌐ A=> C 3, 1, H.S.

5. A v C 4, Impl.

6. ≵ C v A 5, Com.

9

1. (A v B) => C /≵ [(C v D => E] => (A => E)

2. (C v D) => E /≵ A => E C.P.

3. A /≵ E C.P.

4. A v B 3, Add.

5. C 1, 4, M.P.

6. C v D 5, Add.

7. ∴ E 2, 6, M.P.

II Here are some arguments, which can be proved using indirect method.

1.

1. Av (B ΛC)

2. A => C /∴ C

3. ⌐ C I.P.

4. ⌐A 2, 3, M.T.

5. B Λ C 1, 4, D.S.

6. C 5, Simp.

7. C Λ ⌐ C 6, 3, Conj.

Seventh step involves contradiction; therefore ⌐C is false which means that C is true.

2.

1. (D v E) => (F => G)

2. (⌐ G v H) => (DΛF) /∴ G

3. ⌐ G I.P.

4. ⌐ G v H 3, Add.

5. D Λ F 2, 4, M.P.

6. D 5, Simp.

7. D v E 6, Add.

8. F=> G 1, 7, M.P.

9. ⌐ F 8, 3, M.T.

10. F 5, Simp.

11. F Λ ⌐ F 10, 9, Conj.

Eleventh step is contradiction. Therefore ⌐G is false; which means that G is true.

3.

1. (H => I)Λ(J => K)

2. (I v K) => L

3. ⌐ L /∴ ⌐ (H v J)

4. H v J I.P.

5. I v K 1, 4, C.D.

6. L 2, 5, M.P.

7. L Λ ⌐ L 6, 3, Conj.

7th step involves contradiction. Therefore H v J is false; which means that ⌐ (H v J) is true.

4.

1. (M v N) => (OΛP)

2. (O v Q) => (⌐ R Λ S)

3. (R v T) => (M Λ N) /∴ ⌐ R

4. R I.P.

5. R v T 4, Add.

6. M Λ N 3, 5, M.P.

7. O Λ P 1, 6, M.P.

8. O 7, Simp.

9. O v Q 8, Add.

10. (⌐ R Λ S) 2, 9, M.P.

11. ⌐ R 10, Simp.

12. R Λ ⌐ R 4, 11, Conj.

Twelfth step involves contradiction. Therefore R is false which means that ⌐R is true.

5.

1. (V => ⌐ W) Λ (X =>Y)

2. (⌐W => Z) Λ (Y => ⌐ A)

3. (Z=> ⌐ B) Λ (⌐ A=> C)

4. V Λ X / ∴ ⌐ B Λ C

5. ⌐ (⌐ B Λ C) I.P.

6. B v ⌐ C 5, De.M.

7. ⌐ Z v A 3, 6, D.D.

8. W v ⌐ Y 2, 7, D.D.

9. ⌐V v ⌐X 1, 8, D.D.

10. (V Λ X) Λ (⌐ V v ⌐ X) 4, 9, Conj.

10th Step involves contradiction. Therefore ⌐ (⌐BΛC) is false, which mean that ⌐BΛC is true.

Notes

We can also prove these arguments using formal proof of validity. Consider 3rd argument.

6.

1. (H => I) Λ (J => K)

2. (I v K) => L

3. ⌐L / ⸫ !⌐ (H Λ J)

4. ⌐I Λ ⌐ K 2, 3, M.T.

5. ⌐ I 4, Simp.

6. ⌐ I v ⌐ K 5, Add.

7. ⌐ H v ⌐ J 1, 7, D.D.

8. ⸫ ⌐ (H Λ J) 8, De.M.

When the 3rd argument was solved using IP method, it involved 7 steps, whereas

formal proof required 8 steps. Therefore the former is shorter and preferable.

Now consider the fifth agreement.

7

1. (V => ⌐ W) Λ (X =>Y)

2. (⌐W => Z) Λ (Y => ⌐A)

3. (Z => ⌐ B) Λ (⌐A => C)

4. V Λ X /⸫ ⌐ B Λ C

5. V=> ⌐ W 1, Simp.

6. V 4, Simp.

7. ⌐ W 5, 6, M.P.

8. X => Y 1, Simp.

9. X 4, Simp.

10. Y 8, 9, M.P.

11. ⌐ W => Z 2, Simp.

12. Z 11, 7, M.P.

13. Y => ⌐ A 2, Simp.

14. ⌐ A 13, 10, M.P.

15. Z => ⌐ B 3, Simp.

16. ⌐ B 15, 12, M.P.

17. ⌐A => C 3, Simp.

18. C 17, 14, M.P.

19. ≵ ⌐ B Λ C 16, 18, Conj.

When the 5th argument was solved using I.P. method, it involved 10 steps; whereas formal proof
required 19 steps. Therefore the former is shorter and preferable.

III Using the method of reductio ad absurdum (Indirect Proof) the following are
proved to be tautologies.

1

1 (A => B) v (⌐ A => B)

2 ⌐ {(A => B) v (⌐ A => B)} 1, I. P.

3 ⌐ (A => B) ∧ ⌐ (⌐ A => B) 2, De. M.

4 ⌐ (A => B) 3, Sim.

5 A ∧ ⌐ B 4, De. M.

6 ⌐ (⌐ A => B) 2, Simp.

7 ⌐ (A v B) 6,Impl.

8 A 5, Simp.

9 ⌐A ∧ ⌐ B 7, De. M.

10 ⌐A 9, Simpl.

11 A ∧ ⌐ A 8, 10., Conj.

Eleventh step involves contradiction which means that there is error in the second step, i.e.,
assumption. Therefore the given expression is a tautology.

2.

1 (A => B) v (B => A)

2 ⌐ {(A => B) v (B => A)} 1, I. P.

3 ⌐ (A => B) ∧ ⌐ (B => A) 2, De. M.

4 ⌐ (A => B) 3, Simp.

5 ⌐(⌐ A v B) 4, Impl.

6 ⌐ (B => A) 3, Simp.

7 ⌐(⌐ B v A) 6, Impl.

8 A ∧ ⌐ B 5, De.M.

9 A 8, Simp.

10 B ∧ ⌐ A 7, De.M.

11 ⌐ A 10, Simp.

12 A ∧ ⌐ A 9,11, Cong.

Explanation for this argument is the same as the one given to the previous one.

3

1 (A => B) v (B => C)

2 ⌐ {(A => B) v (B => C)} 1, I. P.

3 ⌐ (A => B) ∧ ⌐ (B => C) 2, De. M.

4 ⌐ (⌐A v B) ∧ ⌐ (⌐ B v C) 3, Impl.

5 (A ∧ ⌐ B) ∧ (B ∧ ⌐ C) 4, De.M.

6 A ∧ ⌐ B 5, Simp.

7 ⌐ B 6, Simp.

8 B ∧ ⌐ C 5, Simp.

9 B 8, Simp.

10 B ∧ ⌐ B 9,7, Conj.

Since nineth step involves contradiction, there is error in the second step.

Therefore our assumption is wrong which means that the first step is a tautology.

4.

1 A v (A => B)

2 ⌐ {A v (A => B)} 1, I. P.

3 ⌐ A ∧ ⌐ (A => B) 2, De. M.

4 ⌐ A ∧ ⌐ (⌐ A v B) 3, Impl.

5 ⌐ A ∧ (A ∧ ⌐ B) 4, De. M.

6 (⌐ A ∧ A) ∧ ⌐ B 5, Ass.

7 ⌐ A ∧ A 6, Simp.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

5.

1 P ≡ ⌐ ⌐ P

2 ⌐ (P ≡ ⌐ ⌐ P) 1, I. P.

3 ⌐ {(P => ⌐ ⌐ P) ∧ (⌐ ⌐ P => P)} 2, Equiv.

4 ⌐ {(P => P) ∧ (P => P) } 3,D.N.

5 ⌐ {(⌐ P v P) v (⌐ P v P)} 4, Impl.

6 (P∧ ⌐ P) ∧ (P∧ ⌐ P) 5, De.M.

In this argument there is contradiction in the last step. Therefore the assumption is false.

Therefore 1 is a tautology.

6

1 ⌐ {(A => ⌐ A) ∧ (⌐ A => A)}

2 ⌐ [⌐ {(A => ⌐ A) ∧ (⌐ A => A)}] 1, I. P.

3 {(A => ⌐ A) ∧ (⌐ A => A)} 2, D. N.

4 (⌐ A v ⌐ A) ∧ (A v A) 3, Impl.

5 ⌐ A v ⌐ A ≡ ⌐ A By Taut.

6 A v A ≡ A By Taut.

7 A ∧ ⌐ A 6,5, Conj.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

7. The next argument is very different.

1 ⌐ {(A => ⌐ A) v (⌐ A => A)}

2 (A => ⌐ A) v (⌐ A => A) 1, I. P.

3 (⌐ A v ⌐ A) v (A v A) 2, Impl.

4 ⌐ A v A By Taut.

It is important to note that the fourth step is not a contradiction. On the other hand, it itself is a tautology. It means that the line no. 1 is itself a contradiction.

V. Truth-table technique and Reductio ad absurdum method - a joint venture:

We can also prove the validity of an argument by integrating the method of reductio ad absurdum with the truth-table technique. We have to make certain assumptions before we use the combination of these two. These assumptions are as follows: 1. All premises are necessarily true. When the premises are truth-functionally compound, the truth-values of components should be such that the compound proposition is necessarily true. 2. The conclusion is necessarily taken to be false. When the conclusion is truth-functionally compound, the truth-values of components should be such that the conclusion is necessarily false. While assigning the truth-values, in accordance with these assumptions, if we discover that any component takes the values 1 and 0 simultaneously, then it means that the path has led us to contradiction. Therefore the assumption that the argument is invalid is false. Hence it must be valid. What is important is that once a certain truth-value is assigned to a component, it becomes a permanent fixture of that component throughout the course of the argument. Let us consider this argument.

1. P1(A => B) => (C Λ⌐ D)

2. P2 (D => E) => F / ∴ ⌐ A => F

Let us assume that (⌐ A => F) = 0

(i.e., it is not the case that ⌐ A => F)

This is possible only when ⌐ A=1 and F=0.

3. In P2 F= 0.

4. P2=1 iff (if and only if)

(D => E) => F

0 1 0

5. (D => E) = 0 iff (D => E)

1 0 0

6. ⌐ D = 0 ∵ D = 1

7. (C Λ⌐ D) = 0 ∵ ⌐ D = 0; and if any one conjunct is false, then the whole

conjunction is false.

8. When (CΛ⌐ D) = 0, which is the consequent, P1 can take the value 1 iff the

antecedent (A => B) = 0 ∵ the consequent is false

9. A = 0 ∵ ⌐ A = 1 (according to the law of contradiction, when A= 0, ⌐ A=1) (See2).

10. A => B necessarily takes the value 1 irrespective of the truth-value of B because

A = 0 (See9).

11. 8 and 10 contradict.

12. (1), i.e., ⌐ (⌐ A => F) = 0 is false

13. ∴⌐ A => F

When P1, P2 and the conclusion are connected properly, it becomes a tautology. In order to get such an expression, implication should connect the conclusion to the premises which in turn are connected with conjunction. Since the method of reductio ad absurdum demands that the conclusion must be assumed to be false when the given argument is valid, the truth-conditions of compound proposition must scrupulously be followed. Therefore if the conclusion is disjunctive, then both the components of the disjunction must be assigned 0-value. On the other hand, if the given conclusion is a conjunction, then it is sufficient if any one compound is assigned the 0-value. Thirdly, if the conclusion is the negation of conjunction, then the conjunction itself must be assigned the value1, which means that both components of the conclusion must take the value-1.

Let us consider an argument in which conclusion is a conjunction.

2

P1 (B v ⌐ A) => (⌐ C Λ D)

P2 (D v E) => ⌐ F / ∴ (A Λ ⌐ F)

1. Let us assume that (A Λ ⌐ F) = 0
2. Out of three instances in which any conjunction is false, let us consider first instance.
3. The conclusion is false when A = 0 and ⌐ F = 0
4. P2 is true iff D v E is false
5. D v E = 0 iff D = 0 and E = 0
6. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
7. P1 is true iff (B v ⌐ A) = 0 (from 6)
8. ∴ ⌐ A = 0 (from 7)
9. 3 and 8 violate the law of contradiction because both A and ⌐ A cannot be false
simultaneously.

10. ∴ A Λ ⌐ F

However, if we consider second instance in which we assume that A = 1 and ⌐ F = 0,

then we get different result.

1. A = 1 and ⌐ F = 0
2. P2 = 1 iff D v E = 0
3. D v E = 0 iff D = 0 and E = 0
4. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
5. P1 = 1 iff (B v ⌐ A) = 0 (from 4)
6. ⌐ A must be 0
7. A = 1 if ⌐ A = 0
8. 7 and 1 are compatible ∵ when A =1, ⌐ A must be 0.
9. ∴ A and ⌐ F = 0

Since in one instance our assumption is wrong and in second instance it is correct, this argument is neither tautological nor contradictory. An argument is said to be contingent when in at least one instance it is true and in atleast one instance it is false. Therefore this argument is called contingent and to arrive at this conclusion we need not consider the result of third circumstance. Therefore it is invalid and to confirm the status of this type of argument at least two instances are necessary. (The student is advised to consider the third instance in which the conclusion is assumed to be false and then work out the problem.) It is evident that the method of reductio ad absurdum, when applied to conjunctive conclusion, makes the construction of proof lengthy which renders it the last choice. Secondly, this method succeeds in showing that the truth-table method is primitive because it can be easily shown that ultimately, any other method directly receives support from the truth-table method. It may be noted the rules of inference and replacement derive their authority from truth-table method only. Consider the rule of C.D. which is of the form {(p => q) Λ (r => s) Λ (p v r)} => (q v s). We shall construct the truth-table to show that this is a tautology.

1	2	3	4	5	6	7	8	9	10	11	12
Sl. No.	p	q	r	s	{(p => q)	Λ	(r => s)	Λ	(p v r)}	=>	(q v s)
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	0	0	1	0
3	1	1	1	0	1	0	0	0	1	1	1
4	1	0	1	1	0	0	1	0	1	1	1
5	0	1	1	1	1	1	1	1	1	1	1
6	1	1	0	0	1	1	1	1	1	1	1
7	1	0	0	1	0	0	1	0	1	1	0
8	1	0	0	0	0	0	1	0	1	1	0
9	0	1	0	0	1	1	1	0	0	1	1
10	1	0	1	0	0	0	0	0	1	1	0
11	0	1	1	0	1	0	0	0	1	1	1
12	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	1	1	1	1	0	0	1	1
14	0	0	1	0	1	0	0	0	1	1	0
15	1	1	0	1	1	1	1	1	1	1	1
16	0	1	0	1	1	1	1	0	0	1	1

In the truth-table method the implication which precedes the consequence component is called the main column. In this table the 11th column is the main column. We notice that in this column, in all 16 instances the truth-value is 1. Therefore the rule is a tautology. Reductio ad absurdum method makes another critical point more than obvious. If any argument is tautological, then it is logically impossible to assign the truth-values (without landing in self-contradiction) in such a way that the conjunction of premises takes the value 1 while the conclusion takes the value 0. It shows that the truth-values cannot be assigned in a random manner to the components of the statements which constitutes the argument.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is the advantage of Indirect Proof? Substantiate your answers.

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2. Briefly explain the difference between the rule of conditioned proof and the strengthened rule.

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3. What is the specialty of combining truth-table method with reductio ad absurdum? Construct an argument using symbols and by applying the methods of truth-table and Reductio and absurdum show that it is a tautology.

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12.11 LET US SUM UP

When the conclusion is conditional, formal method does not help. There are three types of conditional statements. There are two kinds of rules of C.P. Indirect Proof is not new to mathematics. Here we reason out in reverse direction. Strengthened rule makes the conclusion independent of assumption.

12.12 KEY WORDS

Reductio ad absurdum: Reductio ad absurdum (Latin: reduction to the absurd) is a form of argument in which a proposition is disproved by assuming the opposite of what is to be proved and deducing its implications to absurd , i.e., self-contradictory consequence.

Tautology: A tautology is a series of statements connected logically which is true in all instances. Contradiction: it is a form of statement which is false in all instances or whose truth table will have only false substitution instances. Contingent statements will have both true and false substitution instances in its truth table.

12.13 QUESTIONS FOR REVIEW

1. What is the advantage of Indirect Proof? Substantiate your answers.
2. Briefly explain the difference between the rule of conditioned proof and the strengthened rule.
3. What is the specialty of combining truth-table method with Reductio ad absurdum? Construct an argument using symbols and by applying the methods of truth-table and Reductio and absurdum show that it is a tautology.

12.14 SUGGESTED READINGS AND REFERENCES

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12.15 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress

1. When there are more statements, the formal method of proof becomes unwieldy. In such circumstances I.P. provides a shorter route, sometimes providing proof in one line.

Notes

2. When the rule of C.P. is applied, always the antecedent of the conclusion is assumed. However when strengthened rule of C.P. is applied this restriction vanishes. Secondly when the antecedent of the conclusion is assumed invariably it has to be justified by writing C.P. on the R.H.S. adjacent to it. On the other hand, in the case of the strengthened rule a bent arrow is used, the extended part of which marks the limits of assumption. The arrow is a substitute for writing C.P.

3. to self-contradiction. When the truth-table method is applied, we proceed from premises to the conclusion. However, when it is combined with I.P. in order to show that the argument is valid, we proceed from the conclusion and assign ‘0’ value and we proceed to show that it leads

UNIT 13: ATTRIBUTES OF THE ATTRIBUTES

STRUCTURE

13.0 OBJECTIVES

After finishing up this unit we can able to understand:

- To know about Attributes of relations
- To identity and definite description
- To make difference Between Attributes, Skills, and Traits

13.1 INTRODUCTION

Investigations of consumer behavior often have as their focus the process by which individuals react to consumption-relevant stimuli and the variance in their reactions, both on an intra and inter-individual basis. Reactions to stimuli may be cognitive and/or behavioral in nature. This paper presents a conceptual framework of some dimensions relevant to the comprehension and interpretation of stimuli. The ideas composing this framework are drawn in large part from the work of cognitive and cross-cultural psychologists involved in investigations of subjective meaning.

It has been found, for instance, that in different cultures, certain "universal" concepts such as "education" and "freedom" can evoke greatly different responses in terms of their meaning to the individual. These differences in concept meaning have been found present within subcultures and ethnic minorities, as well. Such findings suggest that even very salient notions well-known to most adults may exhibit a high variance in meaning for individuals coming from different socioeconomic backgrounds or having different cultural experiences (Szalay and Deese 1978).

The theories and conjectures which have been advanced by behavioral researchers involved in the investigation of meaning and its subjective variance may be quite useful to consumer researchers. Specifically, they may prove especially valuable in furthering our understanding of how consumers assign meaning to products, from what sources they derive information useful in assigning meaning and, importantly, how differences in product meaning may arise among individuals. Answers to questions of this type may take us a great distance toward comprehending the cognitive underpinnings of consumers' perceptions and preferences in such areas as music, the visual arts and product design. Conceptual modeling of consumer response to such products as jazz records, clothing styles, art objects, motion pictures and restaurants has proven so difficult that few empirical investigations of these product areas have been attempted. This is likely due, at least in part, to the lack of adequate conceptualizations of the cognitive processes and social forces which may influence subjective consumption.

13.2 ATTRIBUTES OF RELATIONS

The focus here is upon various dimensions relevant to consumers' assignment of meaning to a product. A useful background for the discussion can be provided by reviewing some selected studies in psychology, consumer behavior and marketing. One of the earliest research streams in psychology which dealt with subjective variance in perception was based upon affective distortion.

Affective Distortion

That the perceptions of individuals concerning stimulus attributes may be subject to affective distortion has been noted since the early work of Thorndike (1920) concerning the "halo" effect in personnel evaluation. Such distortion may be due to at least two separate processes (Blumberg, De Soto, and Kueth 1966; Burnaska and Hollman 1974; Stanley 1961; Willingham and Jones 1958). Common perceptual distortion arises to the extent that (a) preferences within a group are relatively homogeneous and (b) consensus exists concerning the favorability of certain attributes, so that the average perceptions of those attributes for the group as a whole are raised (lowered) for those objects generally viewed as good (bad).

By contrast, idiosyncratic perceptual distortion occurs when a particular individual's global evaluation of an object pulls his or her own rating of the object on some attribute away from the group's mean rating. Either heterogeneity of preferences across subjects or disagreement concerning the favorability of attributes is required in order for idiosyncratic distortion to occur. Its effects may, therefore, be incremental to those of common perceptual distortion, (Holbrook and Huber 1979).

In consumer research, a method to measure the extent of idiosyncratic perceptual bias was pioneered by Beckwith and Lehmann (1975) and has been applied in several subsequent studies (Bemmaor and Huber 1978; James and Carter 1978; Moore and James 1978; Johansson, MacLachlan, and Yalch 1976; Beckwith and Lehmann 1976). Briefly, this approach regresses each respondent's belief on a given attribute toward each object against both the group's mean belief on that attribute for each object and the respondent's own global evaluation of each object. The relative size and significance of the resulting beta coefficients are taken as measures of the degree of halo effect: the larger the relative role of an individual's global evaluation in predicting his belief, the greater the assumed perceptual distortion.

Holbrook and Huber (1979) point out that the Beckwith-Lehmann procedure, as applied in consumer research, typically focuses exclusively on the individual level and does not take into account common perceptual distortion. These two researchers, following the lead of Beckwith and Kubilius (1978) have developed a general approach for separating perceptual dimensions from affective overtones (Holbrook and Huber 1979), and demonstrated its application to the area of jazz recordings.

Affective Distortion in Consumer Research

While the notion of affective distortion has been a valuable one in personnel research where it may be necessary for legal or ethical reasons to segregate rater affect from objective evaluations of worker performance, it may be not as applicable a conceptual tool in consumer research. This is because there is a potential danger that removing the affective component from perception may distort the meaning which a product has for the consumer, and provide an incomplete and possibly misleading picture of the consumer's response to it. Attempts to purge perceptual responses of affective overtones may be due to desires of researchers to more accurately measure the "real" meaning of the product to the consumer. However, Szalay and Deese (1978) argue that the true psychological meaning of a stimulus to an individual necessarily includes affect as a part of its totality. Desires to remove this affective component may be caused by the researcher's belief that the stimulus must be "stripped down" to its objective meaning. Such beliefs are mistakenly drawn from lexical or philosophical perspectives regarding concept meaning, according to Szalay and Deese (1978), and are inappropriate for discerning psychological meaning.

Szalay and Deese (1978) note that there are three alternative perspectives for viewing the meaning of a stimulus: that of the linguist, the philosopher and the behavioral scientist. The interest of the linguist centers upon lexical meaning, that is the "conventional and arbitrary relation between a word and its referent" (Szalay and Deese 1978). The basis of lexical meaning is convention. Lexical meaning has its roots in the use of language by the individuals in a society. It focuses upon habits of language and their correlated mental processes. As Szalay and Deese note, "... Lexical meaning is inappropriate for application to psychological processes in individual human beings..." as it does not represent subjective meaning, (1978 p. 2).

The philosophical perspective of meaning centers upon the concept-referent relationship. This relationship has been of great interest to contemporary cognitive theorists and is essentially rational and logical in nature. With this perspective, meaning becomes synonymous with factual knowledge. This emphasis leads to an epistemological interest in meaning and concern with problems intrinsic to the acquisition and transfer of knowledge, (Szalay and Deese 1978).

The psychological meaning of a concept, however, is that most relevant to many consumer behavior applications. As Szalay and Deese (1978, p.2) state "Psychological meaning describes a person's subjective perception and effective reactions ... It characterizes those aspects that are most salient in an individual's reactions and describes the degree and direction of affectivity ... Logical and Linguistic analysis creates a natural disposition to neglect what is important in psychological meaning, the fact that certain components are more central to psychological representation than others and that ... psychological meaning is suffused with affectivity."

One of the earliest behavioral scientists to investigate psychological meaning as a set of varied dimensions was Osgood (1952). Despite the fact that several criticisms have been leveled at Osgood's research tool for examining meaning, the semantic differential, there is general acceptance of his notion that "meaning is a bundle of components." These components represent the main constituents of the individual's understanding and evaluation of a concept. They may represent experiences, images, information and feelings concerning the concept which has been accumulated directly on vicariously over time (Szalay and Deese 1978). Thus to separate "affective distortion" from the consumer's perception of a stimulus would seem to be a misguided activity. Affect does not represent a distortion of concept meaning, but rather is a vital portion of that meaning.

Factual vs. Evaluative Content

Another area which has recently been cited as having substantial importance in the meaning which consumers assign to products is whether the information they receive concerning the product is "factual" or "evaluative" in nature, (Holbrook 1978). Holbrook has used such an approach to investigate advertising messages on consumer perceptions of automobiles. He states the conceptual underpinnings of this perspective as follows (1978, p. 547):

"One fundamental dimension of verbal content, based on its semantic properties, is the degree to which a message is predominantly factual or evaluative. This basic distinction between two general types of meaning has been emphasized in a wide range of disciplines as diverse as philosophy ("referential" vs. "emotive" meaning or "designative" vs. "appraisive" meaning), aesthetics ("formalist" vs. "expressivist" meaning), linguistics ("cognitive" vs. "affective" components), and psycholinguistics ("representational" vs. "emotive" processes, "symbolic" vs. "evocative" functions or "denotative" vs. "connotative" meaning)."

Holbrook (1978, p. 547) states that factual content may be defined as "logical, objectively verifiable descriptions of tangible product features"; in contrast, evaluative content might consist of "emotional, subjective impressions of intangible aspects of the product." One problem with this approach, however, is that it may group together alternative types of meaning which are not necessarily correlated; and treat as separate some types of meaning which may be related. It has been found, for instance, that consumers may attach emotional responses to tangible, objective product features. That is, the consumer may feel emotionally distressed if he/she is exposed to certain tangible, but personally offensive, product attributes. Thus, objective (i.e. tangible) attributes may inspire affective responses.

Similarly, a product may have associated with it certain intangible features to which the consumer attaches no emotional response. For example, the book, Philosophy of Social Science, by Richard Rudner (1966) deals with issues concerning the construction and application of theories about social, economic, political and psychological phenomena, certainly a collection of intangible notions; yet the scholar who "consumes" this book may have no emotional reaction to it, only an intellectual reaction.

Finally, it may be an important conceptual misnomer to term only intangible product attributes as evaluative and tangible product attributes as factual. That an attribute is factual, that it is an objectively verifiable property of the stimulus, does not preclude it from being evaluated. Indeed, consumer preferences and value judgments for some products may center on tangible product attributes such as miles per gallon, color, size, leather versus plastic, and so forth. Thus, the evaluation of a stimulus attribute would seem to be a dimension independent of whether that attribute is tangible or intangible.

Functional vs. Aesthetic Attributes

A third area of relevance was addressed recently in some innovative research by Sewall (1978a, 1978b). Sewall investigated consumer perceptions and preferences for a product class in which functional attributes, (as he termed attributes such as product ingredients, price and size) were held constant while "aesthetic attributes" of style and color were varied. The product class used in his research was that of bed linens, but Sewall appropriately notes that the research issue extends to a variety of other product types in which style and design significantly influence consumer demand. In this regard Sewall (1978 p.65) cites "home furnishings (drapes, rugs, sofas, wallpaper), clothing (suits, dresses, t-shirts), and gift items (wrapping paper, greeting cards)."

In such product classes an important set of determinants for consumer perception and preference is what he terms the "aesthetic" attributes associated with a particular design, pattern or color. Sewall (1978a) notes that the perceived subjective meaning of such aesthetic attributes may be either heterogeneous or homogeneous across diverse consumer segments and that consumer preferences for these attributes may also vary greatly among segments or be held in common. For example, because of cultural conditioning or other reasons most Americans may report that Jane Fonda is 'politically radical,' a Cadillac is 'prestigious,' and the movie Jaws was 'frightening'; yet preferences for products which are radical, prestigious or frightening may vary widely among consumers. Further, such preferences may likely interact with the product class under consideration. For one product class (for example a discotheque) a high level of excitement may be preferred, while in another (for example luggage) a low level of excitement may be considered preferable.

ATTRIBUTE DIMENSIONS AND PRODUCT MEANING

The relevant aspects of a product stimulus and their role in creating meaning appear to revolve around at least three dimensions: (1) tangibility, (2) perception and, (3) evaluation. The first dimension, tangibility, arises directly from the stimulus, itself. The second and third dimensions, perception and evaluation, refer to cognitive responses generated in the individual concerning the stimulus.

A model of possible linkages between these three dimensions is presented below in Figure One. As is indicated by this conceptualization, attributes may be dichotomized into tangible and intangible features which are associated with the stimulus. This stimulus attributes influence individual perceptual processes and may give rise to response variance, both of a common and idiosyncratic nature. Perceptual processes concerning the product are viewed as one influence upon individual evaluative processes. The rationale and theoretical utility of the first two portions of this conceptualization will now be discussed. The nature of the evaluative process is too complex to be addressed in the present paper. However, it is discussed in detail in Hirschman (1979).

STIMULUS ATTRIBUTE TANGIBILITY

Tangible Attributes

There is a dichotomy of stimulus attributes which may be based upon their tangibility. Tangibility means that an attribute is accessible through the senses, it is palpable. With regard to a product stimulus, a tangible attribute is one which arises directly from the product and may be detected by the individual through one or more of the five senses. Hence, product attributes which may be seen, touched, heard, tasted or smelled are tangible attributes. Such attributes are objective characteristics of a product because they exist independent of the mind and are derived from sensory Perception.

Garner (1978) has noted that tangible features of a stimulus may be grouped into three categories: dichotomous (presence/absence), multichotomous and multi-leveled. The first of these categories, dichotomous, refers to attributes which may be present or absent in a given stimulus, and, if present, have only one level or value. An example of such a stimulus attribute is that of a pollution control valve in an automobile. The automobile either has such a device or it does not.

The second category is that of an attribute multichotomy. In this instance, the product feature is always present, but assumes only one of several possible values. The values are not ordered, but rather are nominal in nature. For example, an automobile may come in any of a variety of colors. Although a selection of possible colors is potentially available, the automobile may assume only one color (or one set of colors) at a given point in time. Further, the automobile will always have color as a characteristic; that is, color will always be an attribute of the product, although its value may be altered at different points in time.

A multi-leveled stimulus is one which assumes a hierarchical distribution of values. That is, one value of the attribute may be ranked as higher or lower than another value of that same attribute. Such attributes are interval or metric in nature and may constitute either continuous or discrete distributions. For example, the horsepower provided by an automobile engine would be an example of a continuously distributed, metrically scaled product attribute; while the number of cylinders in an automobile engine is a discretely distributed, metrically scaled attribute. Often, the distributions of such attributes are step-like or of limited range. For example, most automobile cylinder values are step-like distributions with a limited range (i.e., 2, 4, 6 or 8).

While these three categories constitute the major ways of classifying tangible product attributes, they are not an exhaustive typology. Two possible additions include those attributes which may be present or absent and, if present, assume one of a variety of nominal values - for example, perfumed versus unperfumed deodorants. A second addition is that of product attributes which may be present or absent and, if present, assume one of a variety of interval or metric values - for example, the number of automated teller machines in a bank.

Intangible Attributes

Unlike tangible attributes which are properties of the product, itself, and may be detected via the senses; intangible attributes exist only within the mind of the individual and are mentally rather than physically associated with the product. They are not corporeal or palpable; yet they may be used by consumers to comprehend and classify the product. Intangible attributes are subjective, in nature. That is, they are determined by the mind as the result of experience, they arise from the subject who is observing rather than the object which is being observed.

This very important, substantive difference between tangible and intangible product attributes is illustrated in Figure Two. As shown a tangible product attribute "comes from" the product to the consumer's mind via the senses. In contrast, an intangible product attribute "comes from" the mind of the consumer (the subject) and is projected to the product (the object). Of course, one of the key areas of interest here is the nature of the mental processes giving rise to the intangible attributes which the consumer projects unto the product. A significant and related research question is determining the sources from which consumers derive the intangible attributes which they, in common with others or idiosyncratically, associate with a product stimulus.

One proposition which may be put forward in this regard is that consumers draw commonly-held intangible attributes largely from socialization processes, for example, reference groups, the family, and social institutions such as the mass media, churches, and schools. On the other hand, idiosyncratic intangible attributes which are associated with a product are perhaps more likely to arise from unique personal experiences.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss the Attributes of relations

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13.3 IDENTITY AND DEFINITE DESCRIPTION

Identity Relation: x = y ≡ x is identical to y

Notes

1. Singular sentences Identity sentences:

$s = c$ Ex: Clark Kent is Superman. Non-identity sentences: $s \neq l$ Ex: Dan Quayle is not Jack Kennedy.

2. Quantifier sentences

Somebody in the room is John. No two snowflakes resemble each other.

3. Exceptives and only statements

Ex. 1: Everyone except Alice had fun. $(x)[(Px \cdot x \neq a) \supset (\exists y)(Fy \cdot Hxy)] \cdot (Pa \cdot {\sim}(\exists y)(Fy \cdot Hay))$ A. “Everything except a is G” is a universal sentence making an exception for a, together with the assertion that a is a counterexample. (The subject class is everything but a.) $(x)(x \neq a \supset Gx) \cdot {\sim}Ga$ B. “All F except a are G” is a universal categorical sentence making an exception for a (subject class: everything that is F, but not a), together with the assertion that a is a counterexample: $(x)[(Fx \cdot x \neq a) \supset Gx] \cdot (Fa \cdot {\sim}Ga)$. Nobody except Henry and David went for a swim. Only Henry went swimming.

4. Superlatives Ex:

Usain Bolt is the fastest man. (b is a man, and faster than every man except b) $Mb \cdot (x)[(Mx \cdot x \neq b) \supset Fbx]$

5. Numerical expressions

Ex: Jack has at least two dogs. $(\exists x)(\exists y)[Dx \cdot Hjx \cdot Dy \cdot Hjy \cdot x \neq y]$ Principle: one existential quantifier for each thing; no two are equal. Ex: Jack has at most two dogs. $(x)(y)(z)[(Dx \cdot Dy \cdot Dz \cdot Hjx \cdot Hjy \cdot Hjz) \supset (x = y \vee x = z \vee y = z)]$ Principle: n+1 universal quantifiers; two of them must be the same. Ex: Jack has exactly two dogs. Principle: He has at least two and any other dog Jack has is identical to one of them.

6. Definite Descriptions (phrases beginning with "the")

Ex. The student in the back left corner is asleep. Three claims: 1) Existence: there exists a student in the back left corner 2) Uniqueness: there is at most one student in the back left corner 3) Predication: that student is asleep $(\exists x)(Sx \cdot Bx \cdot (y)((Sy \cdot By) \supset y = x) \cdot Ax)$ Principle: "The φ is Ψ" means: there exists an x that is φ; φy implies y = x; and Ψx.

'The' is the most commonly used word in written English, and for that reason if for no other its formalisation is of great importance. The doctrine that at least a large and central class of its uses can indeed be captured in predicate logic with identity is due to Bertrand Russell and is known as the theory of definite descriptions. The theory has philosophical ramifications, raising problems which are central to philosophical logic and the philosophy of language. The purpose of this section is to outline the theory and sketch part of the debate over some of the issues.

A definite description may be defined roughly as a phrase of the form 'the F'. Such phrases are grammatically rather like proper names. At first sight, they seem to mean much the same as proper names too. It appears a stylistic matter whether one refers to the Michail Gorbachev by that name or as 'the last President of the Soviet Union', and logic is indifferent to style. Consider, however, the phrase 'The President of Australia in 1950'. This is clearly meaningful, despite the fact that there was no such individual. In ordinary logic, names are guaranteed to refer to exactly one individual, whereas definite descriptions cannot be idealised in that way since they can be constructed from any predicates whatsoever. Moreover, names have to be assigned to their bearers by an act of naming whereas a definite description has an internal structure which enables us to understand it, discover which thing if any it picks out, investigate claims made using it and the like without our having been made party to any specific convention concerning its reference. Hence there is a vital difference between what it is to grasp the meaning of a name and what it is to grasp the meaning of a definite description. Finally, there is a puzzle concerning identity statements like

China is the most populous country

or (Russell's own example)

Scott is the author of Waverley

which are true, although the substitution of one term for the other may not always preserve truth in contexts which are not truth functional. For example, it is true that George IV wanted to know whether Scott was the author of Waverley, but false that he wanted to know whether Scott was Scott. Again, it is logically provable that China is China, but not that China is the most populous country. Since the use of =E in these contexts does not even preserve truth, the names and the definite descriptions cannot mean the same, even if they refer to the same objects.

Russell's idea (On Denoting, 1905, reprinted in) for explaining these facts is to construe a sentence like

The natural satellite of the Earth is airless

as being really a conjunction. It asserts, says Russell, two things:

(a) The Earth has exactly one natural satellite.
(b) Any natural satellite of the Earth is airless.

These two, and hence their conjunction, can be expressed in the notation of first order logic with identity, providing a solution to the problem of bringing the logical behaviour of 'the' within reach of our formal system. Most neatly:

The F exists = $\exists x \, \forall y \, (Fy \leftrightarrow x = y)$
The F is G = $\exists x \, (\forall y \, (Fy \leftrightarrow x = y) \wedge Gx)$

Notice that the theory does not provide a direct translation of the definite descriptive phrase 'the F', for the above formula contains no part which may be so read. Such phrases are analysed only in context. That is, we are given not a direct equivalent of the definite description but a way of finding, for any sentence in which such a description occurs, an equivalent sentence in which no definite description occurs. Such a means of eliminating a locution as a logical primitive is called a contextual definition.

Russell's theory of definite descriptions therefore meets the first requirement, that the constructions be rendered logically tractable. It also offers solutions to the other motivating problems. It accounts for definite descriptions which fail to refer, by analysing sentences in which they occur as making false claims.

The President of Australia in 1950 had a wooden leg

is analysed as a conjunction

There was exactly one President of Australia in 1950, and anyone who was President of Australia in 1950 had a wooden leg

which is false because its first conjunct is false. One way for it to be false would be for the President to have had two natural legs; but it can turn out false for the alternative reason that there was no president. The Russellian theory allows us to say all this, make the requisite distinctions and so forth, and to represent its logic within the standard first order system.

Secondly, the theory accounts for the way that understanding newly encountered definite descriptions differs from understanding newly encountered names. It amounts to understanding the embedded indefinite descriptions ('... is a satellite', etc.) plus knowing how quantifiers and identity work. This outcome of the theory squares well with common sense. Historically the Russellian analysis of definite descriptions was of great philosophical importance, as it demonstrated that the apparent or "grammatical" form of a sentence could be misleading as to its semantics, or what Russell sees as its real, deep or "logical" form, and that logical analysis could thus reveal hidden truths and solve philosophical problems. The theory might still be wrong, but it is not just silly; and its success in dealing with the stated motivating problems at least places the onus on its critics to produce an alternative account capable of similar success.

For the purposes of discussing theories of definite descriptions it will be convenient to adopt some notation. One way to proceed is to introduce a symbol

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and use this to create terms out of formulae. So

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x Fx is a term which can occur in formulas just like a proper name, read as 'the thing, x, such that Fx' or more briefly as 'the F'. Using this notation, "The F is G" is written

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x Fx).

More generally, for any formula A containing a name m and for any variable v not occurring in it,

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v Avm

is a term. Note that since there may be no such thing as the F, we cannot allow definite descriptive terms in general to take the place of names in rules such as ∀E and =I. In fact, it is not easy to re-work logic to allow such non-denoting terms (though there are ways: see the section below on free logic for one).

Another idea, perhaps closer to Russell's account, and certainly easier from the viewpoint of formal logic, is to use the new symbol not to create terms but to form quantifiers. That is, for instance, we write

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$(x: Fx)\ Gx$

to say that the F is G—that is, "For the (one and only) thing x such that Fx, Gx." In general, where A and B are formulae containing a name m, and v is a variable not occurring in either of them,

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$(v: Avm)\ Bvm$

is a formula with the truth conditions given in Russell's theory: it is true for an interpretation iff exactly one thing in the domain satisfies A and it also satisfies B. It is not hard to devise introduction and elimination rules for the new quantifier in such a way that the equivalence

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$(x: Fx)\ Gx \leftrightarrow \exists x\, (\forall y\, (Fy \leftrightarrow x = y) \land Gx)$

is provable as a theorem.

We shall regard this view of definite descriptions as quantifiers as the formal realisation of Russell's own account, and we shall refer to the alternative of construing them as terms which may fail to denote as the "modified Russellian" view. On the modified Russellian view, the analogue of the above equivalence

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$x\ Fx) \leftrightarrow \exists x\, (\forall y\, (Fy \leftrightarrow x = y) \land Gx)$

(at least for "appropriate" G, somehow defined) gives the truth conditions of the atom containing the definite description, but that description remains a genuine singular term doing the semantic work of picking out at most one individual.

A third account of definite descriptions, which is sometimes used in practical applications and which short-circuits the whole difficulty of non-referring terms, differs from the modified Russellian one in that where there is no unique F, the term

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x Fx is taken to refer not to the F, there being no such thing, but to some designated "junk" object—say, the number zero, or the empty set, or a special "null" thing introduced for the purpose. This guarantees successful reference in every case, so that the usual recursive calculation of truth conditions can get under way. Of course, constraints have to be imposed at the level of deduction, for we do not want strange and unnatural mathematics to emerge from the convention that 'the largest integer' designates zero! Such constraints are not difficult to devise, however, and mathematicians are often content with this sort of solution to the problems of vacuous definite descriptions. Philosophically, however, it leaves something to be desired; at the very least, its artificiality grates.

More important from the philosophical perspective is yet another account, deriving from Gottlob Frege (Über Sinn und Bedeutung, 1892, reprinted in) and taken up and promoted by Peter Strawson (On Referring, 1950, reprinted in). On this Frege-Strawson view, just as on the modified Russellian view, vacuous terms like 'The present President of Australia' have no reference at all; they just fail to denote anything.

The difference between Frege-Strawson and modified Russell is that atomic sentences containing such terms are claimed not to be false , but to lack truth value altogether. The resultant formal logic is nicely presented by Timothy Smiley in.

Strawson issues several specific complaints against Russell. He charges that Russell has confused the meaning of a name with the bearer of the name, so that a name without a bearer would be meaningless, that he has confused sentences with the statements they can be used to make, that he has confused entailment with presupposition, and that anyway the entire project of formalising the logic of ordinary language is misconceived. He also claims that the Russellian theory, like the modified Russellian one and the "junk object" one, gives intuitively incorrect results in making 'The President of Australia is tall' false when actually it has no truth value. The intuitions on which this last claim is based can perhaps be set aside for the present; they are not universally shared, and anyway it is a mistake to leap at intuitive judgments in disputable cases like this before the formal issues have been clarified.

It is possible that Russell at one time confused the meanings of names with the objects those names denote. Certainly he believed that for a name (or anything else) to have a meaning it must denote something, though this of course does not convict him of the particularly silly view that the meaning is the thing denoted. Even if Russell did believe wrongly that meaning is denoting, however, this would not be very pertinent to the debate, since later proponents of the theory of descriptions, such as Quine, are not at all confused on that point. The weaker doctrine, that an expression is not really a name unless it denotes something, is enough to force a distinction between logically proper names and mere definite descriptions, though it is open to a defender of the modified Russellian view to hold such a doctrine with respect to proper names and yet to construe definite descriptions as genuine singular terms.

The distinction between indicative sentences and the statements made by their use is fairly straightforward. Consider the sentence 'I am over 21'. This sentence could have been used by me in the year 2000 to state that J. Slaney was over 21 in 2000; a use of the very same sentence by you now, or in 2000, or by me in 1970, would result in the making of a different statement possibly with a different truth value. So one sentence can be used to make many statements. Moreover, one statement can be made by means of many different sentences. 'I am over 21' and 'He will be over 21 five years from now' can both be used to make the same statement, though the two uses have to involve different speakers and times. Now Strawson contends, not implausibly, that whereas meaning is a property of sentences, truth value is a property of statements or of the uses of sentences. So a sentence like 'The President of Australia has passed Logic 1' could, when Australia becomes a republic, be used to make either a true statement or a false one. As things stand, a use of it here and now fails to make any statement and hence does not achieve either truth value. Russell sees only three possibilities for what purports to be a sentence: it is either true, false or meaningless. Strawson can allow that a sentence which is perfectly meaningful may be used in such conditions that it makes neither a true statement nor a false one.

A presupposition of a sentence S is a condition which has to obtain in order for the use of S to succeed in making a statement and hence to have a truth value. For example, the existence of the President of Australia is presupposed by the sentence 'The President of Australia has passed Logic 1'. Russell's mistake, it is held, was to confuse the presuppositions of sentences containing definite descriptions with what those sentences assert or entail. Whether there is such a mistake, and if so whether Russell and his followers are guilty of it, are contentious philosophical questions which cannot be explored further here.

Strawson also charges Russell with overlooking the obvious fact that most uses of the word 'the' in English depend heavily on the context for their contribution to reference. Consider, for example,

A man and a woman joined the crowd in the street. The man gave the woman a small, square package.

Clearly in the second sentence we are not asserting, or presupposing, that there is only one man and one woman in the universe (or indeed in the particular domain of discourse). It seems that much of the analysis of 'the' in English must be concerned with making sense of this kind of utterance. Russell has no theory adequate for such a task. He denies, however, that the analysis of everyday speech is an aim of his theory, contending instead that his formal language is to be added to the resources of English in order to improve it for scientific and like purposes. Just how the formal theories of logic and kindred disciplines like grammar and mathematics are related to natural language is another extremely hard problem which is not to be solved here. In various places in these notes I have tried to suggest and illustrate some of the salient features of this complex relationship as I see it. My hopes have been fulfilled if you have been caused to worry about the matter.

13.4 DIFFERENCE BETWEEN ATTRIBUTES, SKILLS, AND TRAITS

We know an attribute is a quality or characteristic of a person, place, or thing. It's an identifying label that alludes to something inherent about them, like charm or cruelty.

A skill, on the other hand, is generally something that is taught. A person will undergo training to learn or improve a particular skill. These might include calligraphy, computer coding, or car repair.

Meanwhile, a trait is an ingrained characteristic or habit that is difficult to learn or unlearn, like shyness or confidence.

To explore the topic of traits more, take a look at some character trait examples.

Positive Attributes

As you look at people around you or develop a character study for your latest short story or novel, how would you label them? What are their attributes? Will these labels denote positive qualities or characteristics? If so, try one of these attributes on for size:

Achiever	Exciting	Meditative	Pleasant	Sensible
Adaptable	Flexible	Mediator	Positive	Sensitive
Ambitious	Focused	Modest	Practical	Sincere
Balanced	Forgiving	Organized	Proactive	Skilled
Cheerful	Generous	Original	Productive	Solid
Consistent	Genuine	Outgoing	Professional	Sporty
Cooperative	Helpful	Particular	Quality	Thoughtful
Courageous	Insightful	Patient	Quick	Trustworthy
Curious	Interesting	Perceptive	Racy	Understanding
Devoted	Inventive	Personable	Responsible	Warm

Negative Attributes

Every story needs conflict or a villain. Although one of these attributes may not be their identifying characteristic, or trait, it might be one of the markers you'll use to describe them. Let's take a look:

Arrogant	Cynical	Inflexible	Pessimistic	Thoughtless
Belligerent	Deceitful	Intolerant	Pompous	Truculent
Boastful	Detached	Irresponsible	Possessive	Unkind

Boring	Dishonest	Jealous	Quarrelso me	Unpredicta ble
Bossy	Domineeri ng	Lazy	Resentful	Unreliable
Callous	Foolish	Mean	Rude	Untrustwort hy
Careless	Greedy	Moody	Sarcastic	Vague
Compulsi ve	Gullible	Nasty	Selfish	Vain
Cowardly	Impolite	Nervous	Stupid	Vengeful
Cruel	Inconsider ate	Patronizin g	Tactless	Vulgar

Professional Attributes

Finally, some attributes aren't quite so personal. Especially in the workplace, certain attributes are simply matter-of-fact. Maybe one of the characters in your book will meet his enterprising lawyer or efficient book editor. If so, you might want to consider honing in on one of these attributes:

Accountabl e	Dependab le	Focused	Motivate d	Respectful
Adaptable	Determine d	Forgiving	Objective	Scheduled
Authentic	Diligent	Generous	Organize d	Scrupulou s
Broadmind ed	Discipline d	Hardworki ng	Passionat e	Selfless
Caring	Effective	Humble	Patient	Sincere

Notes

Collaborative	Efficient	Innovative	Perseverant	Studious
Consistent	Empathetic	Kind	Planner	Thinker
Courteous	Engaging	Listens well	Precise	Transparent
Credible	Enthusiastic	Loyal	Proactive	Trustworthy
Decisive	Evolving	Methodical	Realistic	Truthful

Attributes Abound

Most of us have more than one attribute. We can be clever and funny, or beautiful and honest. While it's true traits are more singular and identifying, it's interesting to see people's attributes unfold. Whether real-life or fictional, attributes abound for many of us.

If you'd like to dive deeper into aspects of personality types and traits, enjoy these Examples of Personality Traits. There, you'll learn more about the famous Myers-Briggs indicators and it may help you develop your new character with wonderful depth. Happy labeling!

Check Your Progress 2

Note: a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

2. Discuss the identity and definite description.
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   ……………………………………………………………………
   ……………………………………………………………………

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3. What are the differences Between Attributes, Skills, and Traits?

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   ……………………………………………………………………
   ……………………………………………………………………
   ……………………………………………………………………
   ……………………

13.5 LET US SUM UP

This unit examines three sets of attributes currently used to characterize stimulus attributes: (1) affective distortion, (2) evaluative vs. factual content, (3) functionality vs. aesthetic appeal. An argument is presented for categorizing stimulus attributes into two groups: tangible features and intangible associations. Based upon this dichotomy a discussion is presented of subjective and objective meaning and several conjectures are advanced concerning their relative importance in product perception.

13.6 KEY WORDS

Attribute: regard something as being caused by.
"he attributed the firm's success to the efforts of the managing director"

13.7 QUESTIONS FOR REVIEW

4. Discuss the Attributes of relations
5. Discuss the identity and definite description
6. What are the differences Between Attributes, Skills, and Traits?

13.8 SUGGESTED READINGS AND REFERENCES

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13.9 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 13.2

Check Your Progress 2

1. See Section 13.3
2. See Section 13.4

UNIT 14:INTUITIVE SET THEORY

STRUCTURE

14.0 OBJECTIVES

In this unit we first discuss some basic ideas concerning sets and functions. These concepts are fundamental to the study of any branch of mathematics, in particular of algebra. In the last section we discuss some elementary number theory. The primary aim of this section is to assemble a few facts that we will need in the rest of the course. We also hope to give you a glimpse of the elegance of number theory. It is this elegance that led the mathematician Gauss to call number theory the 'queen of mathematics', we would like to repeat that this unit consists of very basic ideas that will be used throughout the course. So go through it carefully.

After reading this unit, you

- should be able to use various operations on sets;
- define Cartesian products of sets;

- check if a relation is an equivalence relation or not, and find equivalence classes;
- define and use different kinds of functions;
- state and use the principle of induction;
- use the division algorithm and unique prime factorization theorem.

14.1 INTRODUCTION

Mathematical topics typically emerge and evolve through interactions among many researchers. Set theory, however, was founded by a single paper in 1874 by Georg Cantor: "On a Property of the Collection of All Real Algebraic Numbers".

Since the 5th century BC, beginning with Greek mathematician Zeno of Elea in the West and early Indian mathematicians in the East, mathematicians had struggled with the concept of infinity. Especially notable is the work of Bernard Bolzano in the first half of the 19th century. Modern understanding of infinity began in 1870–1874 and was motivated by Cantor's work in real analysis. An 1872 meeting between Cantor and Richard Dedekind influenced Cantor's thinking and culminated in Cantor's 1874 paper.

Cantor's work initially polarized the mathematicians of his day. While Karl Weierstrass and Dedekind supported Cantor, Leopold Kronecker, now seen as a founder of mathematical constructivism, did not. Cantorian set theory eventually became widespread, due to the utility of Cantorian concepts, such as one-to-one correspondence among sets, his proof that there are more real numbers than integers, and the "infinity of infinities" ("Cantor's paradise") resulting from the power set operation. This utility of set theory led to the article "Mengenlehre" contributed in 1898 by Arthur Schoenflies to Klein's encyclopedia.

The next wave of excitement in set theory came around 1900, when it was discovered that some interpretations of Cantorian set theory gave rise to several contradictions, called antinomies or paradoxes. Bertrand Russell and Ernst Zermelo independently found the simplest and best known paradox, now called Russell's paradox: consider "the set of all sets that are not members of themselves", which leads to a contradiction since it must be a member of itself and not a member of itself. In 1899 Cantor had himself posed the question "What is the cardinal number of the set of all sets?", and obtained a related paradox. Russell used his paradox as a theme in his 1903 review of continental mathematics in his The Principles of Mathematics.

In 1906 English readers gained the book Theory of Sets of Points by husband and wife William Henry Young and Grace Chisholm Young, published by Cambridge University Press.

The momentum of set theory was such that debate on the paradoxes did not lead to its abandonment. The work of Zermelo in 1908 and the work of Abraham Fraenkel and Thoralf Skolem in 1922 resulted in the set of axioms ZFC, which became the most commonly used set of axioms for set theory. The work of analysts such as Henri Lebesgue demonstrated the great mathematical utility of set theory, which has since become woven into the fabric of modern mathematics. Set theory is commonly used as a foundational system, although in some areas—such as algebraic geometry and algebraic topology—category theory is thought to be a preferred foundation.

This unit introduces set theory, mathematical induction, and formalizes the notion of mathematical functions. The material is mostly elementary. For those of you knew to abstract mathematics elementary does not mean simple (though much of the material is fairly simple). Rather, elementary means that the material requires very little previous education to understand it. Elementary material can be quite challenging and some of the material in this chapter, if not exactly rocket science, may require that you adjust your point of view to understand it. The single most powerful technique in mathematics is to adjust your point of view until the problem you are trying to solve becomes simple. Another point at which this material may diverge from your previous experience is that it will require proof. In standard introductory classes in algebra, trigonometry, and calculus there is currently very little emphasis on the discipline of proof. Proof is, however, the central tool of mathematics. This text is for a course that is a student's formal introduction to tools and methods of proof.

14.2 INTUITIVE SET THEORY

Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used to define nearly all mathematical objects.

The modern study of set theory was initiated by Georg Cantor and Richard Dedekind in the 1870s. After the discovery of paradoxes in naive set theory, such as Russell's paradox, numerous axiom systems were proposed in the early twentieth century, of which the Zermelo–Fraenkel axioms, with or without the axiom of choice, are the best-known.

Set theory is commonly employed as a foundational system for mathematics, particularly in the form of Zermelo–Fraenkel set theory with the axiom of choice. Beyond its foundational role, set theory is a branch of mathematics in its own right, with an active research community. Contemporary research into set theory includes a diverse collection of topics, ranging from the structure of the real number line to the study of the consistency of large cardinals.

You must have use the tile word 'set' off and on in your conversations to describe any collection. In mathematics the term set is used to describe any well-defined collection of objects, that is, every set should be so described that given any object it should be clear whether the given object belongs to the set or not. For instance, the collection N of all natural numbers is well defined, and hence is a set. But the collection of all rich people is not a set, because there is no way of deciding whether a human being is rich or not. There letter, E, denotes 'belong I{,' .s the abbreviation of the Greek If S is a set, an object a in the collection S is called an element of S.

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Set theory begins with a fundamental binary relation between an object o and a set A. If o is a member (or **element**) of A, the notation $o \in A$ is used. A set is described by listing elements separated by commas, or by a characterizing property of its elements, within braces { }. Since sets are objects, the membership relation can relate sets as well.

Notes

A derived binary relation between two sets is the subset relation, also called **set inclusion**. If all the members of set A are also members of set B, then A is a subset of B, denoted $A \subseteq B$. For example, {1, 2} is a subset of {1, 2, 3} , and so is {2} but {1, 4} is not. As insinuated from this definition, a set is a subset of itself. For cases where this possibility is unsuitable or would make sense to be rejected, the term proper subset is defined. A is called a **proper subset** of B if and only if A is a subset of B, but A is not equal to B. Also 1, 2, and 3 are members (elements) of the set {1, 2, 3} but are not subsets of it; and in turn, the subsets, such as {1}, are not members of the set {1, 2, 3}.

Just as arithmetic features binary operations on numbers, set theory features binary operations on sets. The:

- Union of the sets A and B, denoted $A \cup B$, is the set of all objects that are a member of A, or B, or both. The union of {1, 2, 3} and {2, 3, 4} is the set {1, 2, 3, 4} .
- Intersection of the sets A and B, denoted $A \cap B$, is the set of all objects that are members of both A and B. The intersection of {1, 2, 3} and {2, 3, 4} is the set {2, 3} .
- Set difference of U and A, denoted $U \setminus A$, is the set of all members of U that are not members of A. The set difference {1, 2, 3} \ {2, 3, 4} is {1} , while, conversely, the set difference {2, 3, 4} \ {1, 2, 3} is {4} . When A is a subset of U, the set difference $U \setminus A$ is also called the complement of A in U. In this case, if the choice of U is clear from the context, the notation A^c is sometimes used instead of $U \setminus A$, particularly if U is a universal set as in the study of Venn diagrams.
- Symmetric difference of sets A and B, denoted $A \triangle B$ or $A \ominus B$, is the set of all objects that are a member of exactly one of A and B (elements which are in one of the sets, but not in both). For instance, for the sets {1, 2, 3} and {2, 3, 4} , the symmetric difference set is {1, 4} . It is the set difference of the union and the intersection, $(A \cup B) \setminus (A \cap B)$ or $(A \setminus B) \cup (B \setminus A)$.
- Cartesian product of A and B, denoted $A \times B$, is the set whose members are all possible ordered pairs (a, b) where a is a member

of *A* and *b* is a member of *B*. The cartesian product of {1, 2} and {red, white} is {(1, red), (1, white), (2, red), (2, white)}.

- Power set of a set *A* is the set whose members are all of the possible subsets of *A*. For example, the power set of {1, 2} is { {}, {1}, {2}, {1, 2} } .

Some basic sets of central importance are the empty set (the unique set containing no elements; occasionally called the *null set* though this name is ambiguous), the set of natural numbers, and the set of real numbers.

14.2.1 Definition

A set is a collection of distinct objects. This means that {1, 2, 3} is a set but {1, 1, 3} is not because 1 appears twice in the second collection. The second collection is called a multiset. Sets are often specified with curly brace notation. The set of even integers can be written: {2n : n is an integer} The opening and closing curly braces denote a set, 2n specifies the members of the set, the colon says "such that" or "where" and everything following the colon are conditions that explain or refine the membership. All correct mathematics can be spoken in English. The set definition above is spoken "The set of twice n where n is an integer". The only problem with this definition is that we do not yet have a formal definition of the integers. The integers are the set of whole numbers, both positive and negative: {0, ±1, ±2, ±3, . . .}. We now introduce the operations used to manipulate sets, using the opportunity to practice curly brace notation.

Definition 2.1 The empty set is a set containing no objects. It is written as a pair of curly braces with nothing inside {} or by using the symbol ∅. As we shall see, the empty set is a handy object. It is also quite strange. The set of all humans that weigh at least eight tons, for example, is the empty set. Sets whose definition contains a contradiction or impossibility are often empty.

Notes

Definition 2.2 The set membership symbol ∈ is used to say that an object is a member of a set. It has a partner symbol ∈/ which is used to say an object is not in a set.

Definition 2.3 We say two sets are equal if they have exactly the same members.

Definition 2.4 The cardinality of a set is its size. For a finite set, the cardinality of a set is the number of members it contains. In symbolic notation the size of a set S is written |S|. We will deal with the idea of the cardinality of an infinite set later.

Definition 2.5 The intersection of two sets S and T is the collection of all objects that are in both sets. It is written $S \cap T$. Using curly brace notation $S \cap T = \{x : (x \in S) \text{ and } (x \in T)\}$ The symbol and in the above definition is an example of a Boolean or logical operation. It is only true when both the propositions it joins are also true. It has a symbolic equivalent ∧. This lets us write the formal definition of intersection more compactly: $S \cap T = \{x : (x \in S) \wedge (x \in T)\}$

14.2.2 Basic operations and their calculus

Recall that a set is a collection of elements.

Given sets AA and BB , we can define the following operations:

Operation	**Notation**	**Meaning**
Intersection	A∩BA∩B	all elements which are in both AA and BB
Union	A∪BA∪B	all elements which are in either AA or BB (or both)
Difference	A−BA−B	all elements which

		are in AA but not in BB
Complement	A‾‾‾A‾ (or ACAC)	all elements which are not in AA

Example 1:

Let A={1,2,3,4}A={1,2,3,4} and let B={3,4,5,6}B={3,4,5,6} .

Then:

A∩B={3,4}A∩B={3,4}

A∪B={1,2,3,4,5,6}A∪B={1,2,3,4,5,6}

A−B={1,2}A−B={1,2}

AC={all real numbers except 1,2,3 and 4}AC={all real numbers except 1 ,2,3 and 4}

Example 2:

Let A={y,z}A={y,z} and let B={x,y,z}B={x,y,z} .

Then:

A∩B={y,z}A∪B={x,y,z}A−B=∅AC={everything except y and z}

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Intuitive set theory?

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14.3 BINARY, N-ARY FUNCTIONS

Binary

In mathematics, a binary function (also called bivariate function, or function of two variables) is a function that takes two inputs.

Precisely stated, a function {\displaystyle f}f is binary if there exists sets {\displaystyle X,Y,Z}X,Y,Z such that

{\displaystyle \,f\colon X\times Y\rightarrow Z}\,f\colon X\times Y\rightarrow Z
where {\displaystyle X\times Y}X\times Y is the Cartesian product of {\displaystyle X}X and {\displaystyle Y.}Y.

Just as we get a number when two numbers are either added or subtracted or multiplied or are divided. The binary operations associate any two elements of a set. The resultant of the two are in the same set. Binary operations on a set are calculations that combine two elements of the set (called operands) to produce another element of the same set.

The binary operations * on a non-empty set A are functions from A × A to A. The binary operation, *: A × A → A. It is an operation of two elements of the set whose domains and co-domain are in the same set.

Addition, subtraction, multiplication, division, exponential is some of the binary operations.

Properties of Binary Operation

- Closure property: An operation * on a non-empty set A has closure property, if a ∈ A, b ∈ A ⇒ a * b ∈ A.

- Additions are the binary operations on each of the sets of Natural numbers (**N**), Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

The additions on the set of all irrational numbers are not the binary operations.

- Multiplication is a binary operation on each of the sets of Natural numbers (**N**), Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

Multiplication on the set of all irrational numbers is not a binary operation.

- Subtraction is a binary operation on each of the sets of Integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).

Subtraction is not a binary operation on the set of Natural numbers (**N**).

- A division is not a binary operation on the set of Natural numbers (**N**), integer (**Z**), Rational numbers (**Q**), Real Numbers(**R**), Complex number(**C**).
- Exponential operation $(x, y) \rightarrow x^y$ is a binary operation on the set of Natural numbers (**N**) and not on the set of Integers (**Z**).

Types of Binary Operations

Commutative

A binary operation * on a set A is commutative if a * b = b * a, for all (a, b) ∈ A (non-empty set). Let addition be the operating binary operation for a = 8 and b = 9, a + b = 17 = b + a.

Associative

The associative property of binary operations hold if, for a non-empty set A, we can write (a * b) *c = a*(b * c). Suppose **N** be the set of natural numbers and multiplication be the binary operation. Let a = 4, b = 5 c = 6. We can write (a × b) × c = 120 = a × (b × c).

Distributive

Let * and o be two binary operations defined on a non-empty set A. The binary operations are distributive if a*(b o c) = (a * b) o (a * c) or (b o c)*a = (b * a) o (c * a). Consider * to be multiplication and o be subtraction. And a = 2, b = 5, c = 4. Then, a*(b o c) = a × (b − c) = 2 × (5 − 4) = 2. And (a * b) o (a * c) = (a × b) − (a × c) = (2 × 5) − (2 × 4) = 10 − 6 = 2.

Identity

If A be the non-empty set and * be the binary operation on A. An element e is the identity element of a ∈ A, if a * e = a = e * a. If the binary operation is addition(+), e = 0 and for * is multiplication(×), e = 1.
Inverse
If a binary operation * on a set A which satisfies a * b = b * a = e, for all a, b ∈ A. a^{-1} is invertible if for a * b = b * a= e, a^{-1} = b. 1 is invertible when * is multiplication.

Solved Example for You

Problem: Show that division is not a binary operation in **N** nor subtraction in **N**.
Solution: Let a, b ∈ **N**

Case 1: Binary operation * = division(÷)
–: **N** × **N**→**N** given by (a, b) → (a/b) ∉ **N** (as 5/3 ∉ **N**)
Case 2: Binary operation * = Subtraction(−)
–: **N** × **N**→**N** given by (a, b)→ a − b ∉ **N** (as 3 − 2 = 1 ∈ **N** but 2−3 = −1 ∉ **N**).

N-ary function

Definition: (1) A function with exactly n arguments. (2) A function which takes any number of arguments, or a variable number of arguments.

This element specifies an n-ary object, consisting of an n-ary object, a base (or operand), and optional upper and lower limits. Examples of n-ary objects are: ,,, and.

example:: The example below demonstrates an n-ary object in its proper form and XML representation:

```
<m:nary>
 <m:naryPr>
  <m:chr m:val="&#8747;"/>
 </m:naryPr>
 <m:sub>
   <m:r>
    <m:rPr>
     <m:scr m:val="roman"/>
     <m:sty m:val="p"/>
    </m:rPr>
    <m:t>0</m:t>
   </m:r>
 </m:sub>
 <m:sup>
   <m:r>
    <m:rPr>
     <m:scr  m:val="roman"/>
     <m:sty  m:val="p"/>
    </m:rPr>
    <m:t>1</m:t>
   </m:r>
 </m:sup>
 <m:e>
  <m:r>
   <m:t>x</m:t>
  </m:r>
  <m:box>
   <m:boxPr>
    <m:diff  m:val="on"/>
```

```
      </m:boxPr>
      <m:e>
        <m:r>
          <m:t>dx</m:t>
        </m:r>
      </m:e>
    </m:box>
  </m:e>
</m:nary>
```

PARENT ELEMENTS

<deg> (§7.1.2.26); <del> (§2.13.5.12); <den> (§7.1.2.28); <e> (§7.1.2.32); <fName> (§7.1.2.37); <ins> (§2.13.5.20); <lim> (§7.1.2.52); <moveFrom> (§2.13.5.21); <moveTo> (§2.13.5.26); <num> (§7.1.2.75); <oMath> (§7.1.2.77); <sub> (§7.1.2.112); <sup> (§7.1.2.114)

CHILD ELEMENTS	SUBCLAUSE
<e> (Base (Argument))	§7.1.2.32
<naryPr> (n-ary Properties)	§7.1.2.72
<sub> (Subscript (Pre-Sub-Superscript))	§7.1.2.112
<sup> (Superscript (Superscript function))	§7.1.2.114

The following XML Schema fragment defines the contents of this element:

```
<complexType name="CT_Nary">
      <sequence>
      <element name="naryPr" type="CT_NaryPr" minOccurs="0"/>
      <element name="sub" type="CT_OMathArg"/>
      <element name="sup" type="CT_OMathArg"/>
      <element name="e" type="CT_OMathArg"/>
      </sequence>
</complexType>
```

History of Set Theory:

Mathematical topics typically emerge and evolve through interactions among many researchers. Set theory, however, was founded by a single paper in 1874 by Georg Cantor: "On a Property of the Collection of All Real Algebraic Numbers".

Since the 5th century BC, beginning with Greek mathematician Zeno of Elea in the West and early Indian mathematicians in the East, mathematicians had struggled with the concept of infinity. Especially notable is the work of Bernard Bolzano in the first half of the 19th century. Modern understanding of infinity began in 1870–1874 and was motivated by Cantor's work in real analysis. An 1872 meeting between Cantor and Richard Dedekind influenced Cantor's thinking and culminated in Cantor's 1874 paper.

Cantor's work initially polarized the mathematicians of his day. While Karl Weierstrass and Dedekind supported Cantor, Leopold Kronecker, now seen as a founder of mathematical constructivism, did not. Cantorian set theory eventually became widespread, due to the utility of Cantorian concepts, such as one-to-one correspondence among sets, his proof that there are more real numbers than integers, and the "infinity of infinities" ("Cantor's paradise") resulting from the power set operation. This utility of set theory led to the article "Mengenlehre" contributed in 1898 by Arthur Schoenflies to Klein's encyclopedia.

The next wave of excitement in set theory came around 1900, when it was discovered that some interpretations of Cantorian set theory gave rise to several contradictions, called antinomies or paradoxes. Bertrand Russell and Ernst Zermelo independently found the simplest and best known paradox, now called Russell's paradox: consider "the set of all sets that are not members of themselves", which leads to a contradiction since it must be a member of itself and not a member of itself. In 1899 Cantor had himself posed the question "What is the cardinal number of the set of all sets?", and obtained a related paradox. Russell used his paradox as a theme in his 1903 review of continental mathematics in his The Principles of Mathematics.

In 1906 English readers gained the book Theory of Sets of Points by husband and wife William Henry Young and Grace Chisholm Young, published by Cambridge University Press.

The momentum of set theory was such that debate on the paradoxes did not lead to its abandonment. The work of Zermelo in 1908 and the work of Abraham Fraenkel and Thoralf Skolem in 1922 resulted in the set of axioms ZFC, which became the most commonly used set of axioms for set theory. The work of analysts such as Henri Lebesgue demonstrated the great mathematical utility of set theory, which has since become woven into the fabric of modern mathematics. Set theory is commonly used as a foundational system, although in some areas—such as algebraic geometry and algebraic topology—category theory is thought to be a preferred foundation.

Basic concepts and notation

Set theory begins with a fundamental binary relation between an object o and a set A. If o is a **member** (or **element**) of A, the notation $o \in A$ is used. A set is described by listing elements separated by commas, or by a characterizing property of its elements, within braces { }. Since sets are objects, the membership relation can relate sets as well.

A derived binary relation between two sets is the subset relation, also called **set inclusion**. If all the members of set A are also members of set B, then A is a **subset** of B, denoted $A \subseteq B$. For example, {1, 2} is a subset of {1, 2, 3} , and so is {2} but {1, 4} is not. As insinuated from this definition, a set is a subset of itself. For cases where this possibility is unsuitable or would make sense to be rejected, the term **proper subset** is defined. A is called a **proper subset** of B if and only if A is a subset of B, but A is not equal to B. Also 1, 2, and 3 are members (elements) of the set {1, 2, 3} but are not subsets of it; and in turn, the subsets, such as {1}, are not members of the set {1, 2, 3}.

Just as arithmetic features binary operations on numbers, set theory features binary operations on sets. The:

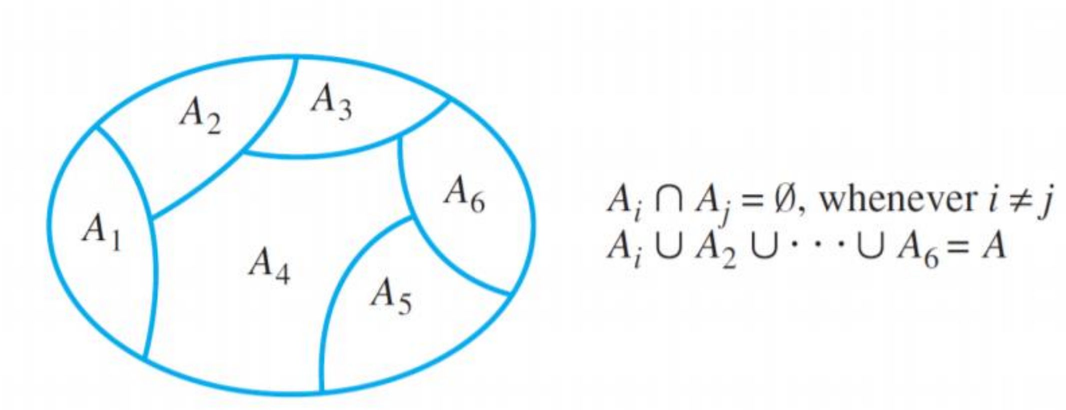
- **Union** of the sets *A* and *B*, denoted $A \cup B$, is the set of all objects that are a member of *A*, or *B*, or both. The union of {1, 2, 3} and {2, 3, 4} is the set {1, 2, 3, 4} .
- **Intersection** of the sets *A* and *B*, denoted $A \cap B$, is the set of all objects that are members of both *A* and *B*. The intersection of {1, 2, 3} and {2, 3, 4} is the set {2, 3} .
- **Set difference** of *U* and *A*, denoted $U \setminus A$, is the set of all members of *U* that are not members of *A*. The set difference {1, 2, 3} \ {2, 3, 4} is {1} , while, conversely, the set difference {2, 3, 4} \ {1, 2, 3} is {4} . When *A* is a subset of *U*, the set difference $U \setminus A$ is also called the **complement** of *A* in *U*. In this case, if the choice of *U* is clear from the context, the notation A^c is sometimes used instead of $U \setminus A$, particularly if *U* is a universal set as in the study of Venn diagrams.
- **Symmetric difference** of sets *A* and *B*, denoted $A \triangle B$ or $A \ominus B$, is the set of all objects that are a member of exactly one of *A* and *B* (elements which are in one of the sets, but not in both). For instance, for the sets {1, 2, 3} and {2, 3, 4} , the symmetric difference set is {1, 4} . It is the set difference of the union and the intersection, $(A \cup B) \setminus (A \cap B)$ or $(A \setminus B) \cup (B \setminus A)$.
- **Cartesian product** of *A* and *B*, denoted $A \times B$, is the set whose members are all possible ordered pairs (a, b) where *a* is a member of *A* and *b* is a member of *B*. The cartesian product of {1, 2} and {red, white} is {(1, red), (1, white), (2, red), (2, white)}.
- **Power set** of a set *A* is the set whose members are all of the possible subsets of *A*. For example, the power set of {1, 2} is { {}, {1}, {2}, {1, 2} } .

Some basic sets of central importance are the empty set (the unique set containing no elements; occasionally called the *null set* though this name is ambiguous), the set of natural numbers, and the set of real numbers.

14.4 EQUIVALENCE AND ORDER RELATIONS

A partition of a set A is a finite or infinite collection of nonempty, mutually disjoint subsets whose union is A. The diagram of Figure 8.3.1 illustrates a partition of a set A by subsets A1, A2, . . . , A6.

Figure 14.1: A Partition of a Set



The fact is that a relation induced by a partition of a set satisfies all three properties: reflexivity, symmetry, and transitivity.

Let A be a set with a partition and let R be the relation induced by the partition. Then R is reflexive, symmetric, and transitive.

Definition of an Equivalence Relation

A relation on a set that satisfies the three properties of reflexivity, symmetry, and transitivity is called an equivalence relation.

Example 2 – An Equivalence Relation on a Set of Subsets Let X be the set of all nonempty subsets of {1, 2, 3}. Then X = {{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}} Define a relation R on X as follows: For all A and B in X, A R B ⇔ the least element of A equals the least element of B. Prove that R is an equivalence relation on X.

– Solution R is reflexive: Suppose A is a nonempty subset of {1, 2, 3}. [We must show that A R A.] It is true to say that the least element of A equals the least element of A. Thus, by definition of R, A R A. R is symmetric: Suppose A and B are nonempty subsets of {1, 2, 3} and A R B. [We must show that B R A.] Since A R B, the least element of A equals the least element of B. But this implies that the least element of B equals the least element of A, and so, by definition of R, B R A.

R is transitive: Suppose A, B, and C is nonempty subsets of {1, 2, 3}, A R B, and B R C. [We must show that A R C.] Since A R B, the least element of A equals the least element of B and since B R C; the least element of B equals the least element of C. Thus the least element of A equals the least element of C, and so, by definition of R, A R C.

Equivalence and Order Section 1:

Equivalence The concept of an equivalence relation on a set is an important descriptive tool in mathematics and computer science. It is not a new concept to us, as “equivalence relation” turns out to be just another name for “partition of a set.” Our emphasis in this section will be slightly different from our previous discussions of partitions in Unit SF. In particular, we shall focus on the basic conditions that a binary relation on a set must satisfy in order to define a partition. This “local” point of view regarding partitions is very helpful in many problems. We start with the definition. Definition 1 (Equivalence relation) An equivalence relation on a set S is a partition K of S. We say that s, t ∈ S are equivalent if and only if they belong to the same block of the partition K. We call a block an equivalence class of the equivalence relation. If the symbol ≡ denotes the equivalence relation, then we write s ≡ t to indicate that s and t are equivalent (in the same block) and s 6≡ t to denote that they are not equivalent. Here’s a trivial equivalence relation that you use all the time. Let S be any set and let all the blocks of the partition have one element.

Two elements of S are equivalent if and only if they are the same. This rather trivial equivalence relation is, of course, denoted by "=". Example 1 (All the equivalence relations on a set) Let S = {a, b, c}. What are the possible equivalence relations on S? Every partition of S corresponds to an equivalence relation, so listing the partitions also lists the equivalence relations. Here they are with the equivalences other than a ≡ a, b ≡ b and c ≡ c, which are always present. {a}, {b}, {c} no others {a}, {b, c} b ≡ c, c ≡ b {b}, {a, c} a ≡ c, c ≡ a {c}, {a, b} a ≡ b, b ≡ a {a, b, c} a ≡ b, b ≡ a, a ≡ c, c ≡ a, b ≡ c, c ≡ b What about the set {a, b, c, d}? There are 15 equivalence relations. For a five element set there are 52. As you can see, the number increases rapidly.

Definition 2 (Binary relation on a set) Given a set S, a binary relation on S is a subset R of S × S. Given a binary relation R, we will write s R t if and only if (s, t) ∈ R. 1 Example 5 (Equivalence relations as binary relations) Suppose ≡ is an equivalence relation on S associated with the partition K. Then the set R = {(s, t) | s ≡ t} ⊆ S ×S is a binary relation on S associated with the equivalence relation. Thus an equivalence relation is a binary relation. The converse need not be true. For example x R y if and only if x < y defines a binary relation on Z, but it is not an equivalence relation because we never have x < x, but an equivalence relation requires x ≡ x for all x. When is a binary relation an equivalence relation? The next theorem provides necessary and sufficient conditions for a binary relation to be an equivalence relation. Verifying the conditions is a sometimes a useful way to prove that some particular situation is an equivalence relation. Theorem 1 (Reflexive, symmetric, transitive) Let S be a set and suppose that we have a binary relation R on S. This binary relation is an equivalence relation if and only if the following three conditions hold. (i) (Reflexive) For all s ∈ S we have s R s. (ii) (Symmetric) For all s, t ∈ S such that s R t we have t R s. (iii) (Transitive) For all r, s, t ∈ S such that r R s and s R t we have r R t. Proof: We first prove that an equivalence relation satisfies (i)–(iii). Suppose that ≡ is an equivalence relation. Since s belongs to whatever block it is in, we have s ≡ s. Since s ≡ t means that s and t belong to the same block, we have s ≡ t if and only if we have t ≡ s. Now suppose that r ≡ s and s ≡ t. Then r and s are in the same block and s and t are in the same block. Thus r and t are in the same block and so r ≡ t.

We now suppose that (i)–(iii) hold and prove that we have an equivalence relation; that is, a partition of the set S. What would the blocks of the partition be? Everything equivalent to a given element should be in the same block. Thus, for each s ∈ S let B(s) be the set of all t ∈ S such that s R t. We must show that the set of these sets form a partition of S; that is, {B(s) | s ∈ S} is a partition of S. In order to have a partition of S, we must have (a) the B(s) are nonempty and every t ∈ S is in some B(s) and (b) for every p, q ∈ S, B(p) and B(q) are either equal or disjoint. Since R is reflexive, s ∈ B(s), proving (a). We now turn our attention to (b). Suppose B(p) ∩ B(q) is not empty. We must prove that B(p) = B(q). Suppose x ∈ B(p) ∩ B(q) and y ∈ B(p). We have, p R x, q R x and p R y. By the symmetric law, x R p. Using transitivity twice: q R x and x R p implies q R p, q R p and p R y implies q R y. By the definition of B, this means y ∈ B(q). Since this is true for all y ∈ B(p), we have proved that B(p) ⊆ B(q). Similarly B(q) ⊆ B(p) and so B(p) = B(q). This proves (b).

The Pigeonhole Principle

We now look at a class of problems that relate to various types of restrictions on equivalence relations. These problems are a part of a much more general and often very difficult branch of mathematics called extremal set theory. The following theorem is a triviality, but its name and some of its applications are interesting. You should be able to prove the theorem. Theorem 2 (Pigeonhole principle) Suppose K is a partition of a set S and |S| = s. If K has fewer than s blocks, then some block must have at least two elements. Where did the name come from? Old style desks often had an array of small horizontal boxes for storing various sorts of papers — unpaid bills, letters, etc. These were called pigeonholes because they often resembled the nesting boxes in pigeon coops. Imagine slips of paper, with one element of S written on each slip. Put the slips into the boxes. At least one box must receive more than one slip if there are more slips than boxes — that's the pigeonhole principle. After the slips are in the boxes, a partition of the set of slips has been defined. (The boxes are the blocks.) Example 7 (Applying the pigeonhole principle) Designate the months of the year by the set numbers M = 12 = {1, 2, . . . , 12}. What is the smallest integer k such that among any k people, there must be at least two people with the same first letter of their last name and same birth month? This is a typical application of the pigeonhole principle. Recall that we can define a partition of a set P by defining a function f with domain P and letting the partition of P be the coimage of f. In this case, we let P be the set of people and define $f : P \to M \times A$, where A is the set of letters in the alphabet, as follows: $f(p) = (m, a)$, where m is the month in which p was born and a is the first letter of p's last name. To be able to apply the pigeonhole principle to obtain the conclusion asked for, we must have $|M \times A| < k$, where $k = |P|$. In other words, we must have $12 \times 26 = 312 < k$. The smallest such k = 313. If k 312 then it is possible to have a group of people, no two of which have the same first letter of their last name and same birth month.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

2. Discuss about Binary and n-nary functions.

   …………………………………………………………………

   …………………………………………………………………

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3. What is Equivalence and order relations?

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14.5 LET US SUM UP

In this unit we have covered the following points.

1) Some properties of sets and subsets.

2) The union, intersection, difference and complements of sets.

3) The Cartesian product of Sets.

4) Relations i-1 general, and equivalence relations in particular.

5) The definition of a function, a 1-1 function, an unto function and a bijective function.

6) The cosmopolitan of functions.

7) The we1.-ordering principle, which states that every subset uf N has a least element.

8) The principals of finite induction, which states that : If P(n) is a statement about some
n E N such that
i) P(1) is true, and
ii) if P(k) is true for some k E N, then P(k+l) is true,
then P(n) is true for every n E N.

9) The principle of finite induction can also be stated as :

If P(n) is a statement about some n E N such that
i) P(l) is true, and
ii) if P(m) is true for every positive integer m < k, then P(k) is true,
then P(n) is true for every n E N.
Note that the well-ordering principle is equivalent to the principle of finite induction.

10) Properties of divisibility in 2, like the division algorithm and unique prime factorization

14.6 KEY WORDS

Binary: In mathematics and digital electronics, a binary number is a number expressed in the base-2 numeral system or binary numeral system, which uses only two symbols: typically "0" and "1". The base-2 numeral system is a positional notation with a radix of 2. Each digit is referred to as a bit

Set Theory: Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics.

14.7 QUESTIONS FOR REVIEW

4. What is Intuitive set theory?
5. Discuss about Binary and n-nary functions.
6. What is Equivalence and order relations?

14.8 SUGGESTED READINGS AND REFERENCES

- Devlin, Keith (1993), The Joy of Sets (2nd ed.), Springer Verlag, ISBN 0-387-94094-4
- Ferreirós, Jose (2007), Labyrinth of Thought: A history of set theory and its role in modern mathematics, Basel: Birkhäuser, ISBN 978-3-7643-8349-7
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- Monk, J. Donald (1969), Introduction to Set Theory, McGraw-Hill Book Company, ISBN 978-0898740066

14.9 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 14.2

Check Your Progress 2

1. See Section 14.3
2. See Section 14.4